DISTANCE FIBONACCI NUMBERS

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By numbers of the Fibonacci type we mean numbers defined recursively by the kth order linear recurrence relation

\[ a_n = b_1 a_{n-1} + b_2 a_{n-2} + \ldots + b_k a_{n-k} \]  \hspace{1cm} (1)

for \( n \geq k \), where \( k \geq 2 \) and \( b_i \geq 0 \), \( i = 1, \ldots, k \), are integers and \( a_0, \ldots, a_{k-1} \) are fixed integers.

For special value \( k \) and \( b_i \), \( i = 1, \ldots, k \) the equality (1) gives the recurrences which define the Fibonacci numbers and their distance generalizations.

1. Fibonacci numbers \( F_n = F_{n-1} + F_{n-2} \), \( n \geq 2 \) with \( F_0 = F_1 = 1 \).

2. \( k \)-generalized Fibonacci numbers (E.P. Miles) 1960

\( f_n = F_{n-1} + f_{n-2} + \ldots + f_{n-k} \), \( k \geq 2 \), \( n > k \) with \( f_i = 0 \) for \( 0 \leq i \leq k-2 \) and \( f_{k-1} = f_k = 1 \).

3. generalized Fibonacci numbers \( F(k, n) \) (M. Kwaśnik, I. Włoch) 2000

\( F(k, n) = F(k, n-1) + F(k, n-k) \), \( k \geq 2 \), \( n \geq k \) with \( F(k, n) = n + 1 \) for \( n = 0, 1, \ldots, k - 1 \)

In the talk we give other types of distance Fibonacci numbers. We show their distinct interpretations also in graphs.

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