ON THE FIBONACCI NUMBERS OF ZYKOV SUMS
OVER PATHS AND CYCLES

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Let $G = (V, E)$ be a graph. A subset $S$ of $V$ is said to be independent whenever for every two different vertices $u, v \in S$ there are no edge between them. The Fibonacci number of $G$, denoted by $\mathcal{F}(G)$, is the total number of independent vertex sets of $G$. This concept was introduced by Tichy and Prodinger in [1]. They proved that $\mathcal{F}(P_n)$ and $\mathcal{F}(C_n)$, the Fibonacci numbers of the path and the cycle of order $n$, are the $(n+2)$-fibonacci number and the $n$-lucas number, respectively.

A circulant graph $C_n[r]$ with consecutive jumps $1, 2, \ldots, r$ is the graph with vertex set $V = \{v_1, v_2, \ldots, v_n\}$ and edge set $E = \{v_i v_{i+j \mod n} : i \in \{1, 2, \ldots, n\}$ and $j \in \{1, 2, \ldots, r\}\}$, where $r \in \mathbb{Z}^+$, and $n \geq r + 1$. In [2], Dosal-Trujillo and Galeana-Sánchez found the Fibonacci numbers of the family of circulant graphs $C_n[r]$ using a recursive method. These numbers are determined by sequences that generalize the Fibonacci and Lucas numbers.

Given a graph $G$ and a family of graphs $\alpha = (\alpha_v)_{v \in V(G)}$ without vertices in common, the Zykov sum $\sigma(G, \alpha)$ is the graph with vertex set $\bigcup_{v \in V(G)} V(\alpha_v)$, and edge set $\bigcup_{v \in V(G)} E(\alpha_v) \cup \{x, y : x \in V(\alpha_u), y \in V(\alpha_v)$ and $uv \in E(G)\}$. In this talk we will show upper and lower bounds of the Fibonacci numbers of the families of Zykov sums, where $G$ are $P_n$ or $C_n$ and for every vertex $v$ in $G$, $\alpha_v$ is a graph of order $r$, $r \in \mathbb{Z}^+$. For these families the Fibonacci numbers once again generalize the Fibonacci and Lucas sequences.

Keywords: independent set, Zykov sums, Fibonacci number of graphs.

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References


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