SOME RESULTS ON THE HEDETNIEMI CONJECTURE

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Hedetniemi conjectured in 1966 that if $G$ and $H$ are finite graphs with chromatic number $n$, then the chromatic number $\chi(G \times H)$ of the direct product of $G$ and $H$ is also $n$. We mention two of the many notable well-known results pertaining to this conjecture and we offer an improvement of the one, and show the bearing it has on the other. The first of these two results is due to Burr, Erdős and Lovász, who showed that when every vertex of a graph $G$ with $\chi(G) = n$ is contained in an $(n-1)$-clique then $\chi(G \times H) = n$. The second, by Duffus, Sands and Woodrow, and obtained independently by Welzl, states that the same is true when $G$ and $H$ are connected graphs with clique number $n-1$. We prove that if $G$ is a graph with $\chi(G) = n$ and is such that the subgraph of $G$ induced by those vertices of $G$ that are not contained in an $(n-1)$-clique is homomorphic to an $n$-critical graph $H$, then $\chi(G \times H) = n$. Our approach will employ a construction of a graph $F$, with chromatic number $n$, that is homomorphic to $G$ and $H$.

Keywords: finite graph, direct product, homomorphism (of graphs), $Ht$-construct, $P$-string, $HPEs$-construct.

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References


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