A tournament $T$ on $n$ vertices is an orientation of the complete graph $K_n$. We say that a tournament $T$ is regular if for every vertex $v$ of $T$, we have that $d^+(v) = d^-(v)$, that is, the outdegree is equal to the indegree at every vertex. As a consequence, a regular tournament has an odd number of vertices $n = 2m + 1$. It is a well-known property that every regular tournament has the same number of directed triangles denoted by $\rightarrow \mathcal{C}_3$.

We consider a 3-uniform hypergraph (a 3-graph for short in what follows) $H_T$ defined by $V(H_T) = V(T)$ and by the set of 3-(hyper)edges $E(H_T) = \{\{x, y, z\} \subseteq V(T) : T\langle x, y, z \rangle \cong \rightarrow \mathcal{C}_3\}$, where $T\langle S \rangle$ is the induced subtournament of $T$ by the subset $S \subseteq V(T)$ (see [3] for details). The heterochromatic (or rainbow) number of 3-graph $H$ (denoted by $h_c(H)$) was introduced in [1] as the minimum number of colors $r$ such that every proper $r$-coloring of the vertices of $H$ leaves at least one rainbow 3-edge. If $h_c(H) = 3$, the 3-graph is said to be tight. In this setting, the heterochromatic number $h_c(T)$ of a tournament $T$ is the heterochromatic number of its associated 3-graph $H$.

In [2], we proved that $h_c(\rightarrow \mathcal{C}_p(J)) = 3$ for every odd prime number $p$, where $\rightarrow \mathcal{C}_p(J)$ denotes a circulant tournament of order $p$ and symbol set $J$. We recall that a circulant tournament $\rightarrow \mathcal{C}_{2m+1}(J)$ is a regular tournament defined by $V(\rightarrow \mathcal{C}_{2m+1}(J)) = \mathbb{Z}_{2m+1}$, the symbol set is $J \subseteq \mathbb{Z}_{2m+1} \setminus \{0\}$ such that for every $j \in \mathbb{Z}_{2m+1} \setminus \{0\}$, we have that $|\{j, -j\} \cap J| = 1$ and $i \rightarrow j$ is an arc of $\rightarrow \mathcal{C}_{2m+1}(J)$ if $j - i \in J$ (modulo $2m + 1$).

In this talk, we exhibit a characterization of tight regular tournaments, that is, those tournaments $T$ for which $h_c(T) = 3$. We also present some properties of the $h_c(T)$-colorings of a regular tournament $T$.

Keywords: heterochromatic number, regular tournaments, 3-uniform hypergraphs.

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References


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