DOMINATING AND COVERING SEQUENCES IN
GRAPHS AND HYPERGRAPHS

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Let \( S = (v_1, \ldots, v_k) \) be a sequence of distinct vertices of a graph \( G \). The sequence \( S \) is a legal closed (respectively open) neighborhood sequence if \( N[v_i] \setminus \bigcup_{j=1}^{i-1} N[v_j] \neq \emptyset \) (respectively \( N(v_i) \setminus \bigcup_{j=1}^{i-1} N(v_j) \neq \emptyset \)) holds for every \( i \in \{2, \ldots, k\} \). If, in addition, the resulting set is a (total) dominating set of \( G \), then we call \( S \) a (total) dominating sequence of \( G \). The maximum length of a (total) dominating sequence is called the Grundy (total) domination number of a graph \( G \).

In a more general framework of covering sequences in hypergraphs it was proven that Grundy Covering Number Problem in hypergraphs is NP-complete, which was an important step in the proof of the NP-completeness of Grundy Domination Number Problem in graphs \([1]\). In this talk, we present a similar result for the total version of Grundy domination number, and show that Grundy Total Domination Number Problem is NP-complete even in bipartite graphs, by using a different approach through the incidence graph of a hypergraph. Next, we discuss the bounds on Grundy total domination number with respect to Grundy domination number. At the end, we consider also the sequences that arise from the domination game, as introduced in \([2]\). As it turns out, the problem is even harder, and we show that verifying whether the game domination number of a graph is bounded by a given integer is PSPACE-complete.

Keywords: Grundy domination number, Grundy covering number, graph algorithm.

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References
