GENERALIZING BONDY’S THEOREM ON SUFFICIENT CONDITIONS FOR HAMILTONIAN PROPERTIES

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In 1980 Bondy [1] proved that a \((k + s)\)-connected graph of order \(p \geq 3\) is traceable \((s = -1)\) or hamiltonian \((s = 0)\) or hamiltonian-connected \((s = 1)\) if the degree sum of every set of \(k + 1\) pairwise nonadjacent vertices is at least \(\frac{1}{2}((k + 1)(p + s - 1) + 1)\). We show that one can allow exceptional stable \((k + 1)\)-sets violating this condition but still implying the considered property. We give upper bounds for the number of such ‘bad’ sets depending on the connectivity and the minimum degree of the graph.

Moreover, we ask for the tightness of the bound in Bondy’s Theorem. It turns out that the bound is not tight for \(k \geq 2\). We improve this sufficient degree condition and show the general tightness of the result.

Keywords: Hamiltonian, hamiltonian-connected, traceable, Bondy’s Theorem, degree sum conditions.

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References