ON A GENERALIZATION OF INTERVAL EDGE-COLORINGS OF GRAPHS

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An interval \((t, h)\)-coloring \([1, 2]\) \((h \in \mathbb{Z}_+)\) of a graph \(G\) is a proper edge-coloring \(\alpha\) of \(G\) with colors \(1, \ldots, t\) such that all colors are used, and the colors of edges incident to each vertex \(v\) satisfy the condition \(d_G(v) - 1 \leq \max S(v, \alpha) - \min S(v, \alpha) \leq d_G(v) + h - 1\), where \(S(v, \alpha)\) is the set of colors of edges incident to \(v\) in the coloring \(\alpha\). For \(h \in \mathbb{Z}_+\), let \(\mathcal{N}^h\) denote the set of graphs which have an interval \((t, h)\)-coloring for some positive integer \(t\). If \(h = 0\), then an interval \((t, 0)\)-coloring is an interval \(t\)-coloring and \(\mathcal{N}^0 = \mathcal{N}\).

In this paper we prove that \(G \in \mathcal{N}^h\) if and only if \(G \Box Q_h \in \mathcal{N}\). On the other hand, for every \(h \in \mathbb{N}\), there exists a connected graph \(G\) such that \(G / \in \mathcal{N}^h\). Next, we consider planar Cartesian products and prove that if \(G\) is a triangle-free outerplanar graph, then \(G \in \mathcal{N}^1\); thus \(G \Box K_2 \in \mathcal{N}\). Also, we show that if \(G\) is a 2-connected outerplanar graph with \(\Delta(G) = 3\), then \(G\) has an interval coloring with no more than 4 colors. Finally, we give a negative answer to the question of Axenovich on interval colorings of outerplanar triangulations.

Keywords: interval coloring, Cartesian product, outerplanar graph.

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References
