ON GRAPHS ADMITTING TWO DISJOINT MAXIMUM INDEPENDENT SETS

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An independent set $S$ in a graph $G$ is maximal if it is not a proper subset of any independent set, while $S$ is maximum if it has a maximum size, denoted as $\alpha(G)$. The problem of whether a graph has a pair of disjoint maximal independent sets was introduced by Claude Berge in early 70’s. Further, this topic was studied in [1, 3, 4].

In this paper, we are focused on finding conditions ensuring existence of two disjoint maximum independent sets.

**Theorem 1** The graph $G$ has two disjoint maximum independent sets if and only if there exists a matching $M$ of size $\alpha(G)$ such that $G[V(M)]$ is a bipartite graph, where $V(M)$ is the set of vertices covered by $M$.

Let $v \in V(G)$. If for every independent set $S$ of $G - N[v]$, there exists some $u \in N(v)$ such that $S \cup \{u\}$ is independent, then $v$ is a shedding vertex of $G$ [5]. Let $\text{Shed}(G)$ denote the set of all shedding vertices.

**Theorem 2** If $G$ has a maximum independent set $S \subseteq \text{Shed}(G)$, then $|\Omega(G)| \geq 2^{\alpha(G)}$, while some $I \in \Omega(G)$ is disjoint from $S$, where $\Omega(G)$ is the family of all maximum independent sets.

If $\alpha(G) + \mu(G) = |V(G)|$, then $G$ is a König-Egerváry graph, where $\mu(G)$ is the size of a largest matching.

**Theorem 3** $G$ is a König-Egerváry graph with two disjoint maximum independent sets if and only if it is a bipartite graph having a perfect matching.
A graph $G$ is unicyclic if it is connected and has a unique cycle.

**Theorem 4** A unicyclic graph $G$ has two disjoint maximum independent sets if and only if, either $G$ is a bipartite graph with a perfect matching, or there is a vertex $v$ belonging to its unique cycle, such that $G - v$ is a forest with a perfect matching.

**Corollary 5** One can decide in polynomial time whether a unicyclic graph has two disjoint maximum independent sets.

It is known that the decision problem whether there are two disjoint maximal independent sets in a graph is **NP-complete** [2].

**Conjecture 6** It is **NP-complete** to recognize graphs with two disjoint maximum independent sets.

**Keywords:** independent set, shedding vertex, König-Egerváry graph.

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**References**


