A \textit{star edge-coloring} of a graph is a proper edge-coloring with forbidden bichromatic paths and cycles of length 4. The least number of colors that suffice for an edge-coloring of a graph $G$ is called the \textit{star chromatic index}, denoted $\chi'_{st}(G)$. The study of this notion was motivated by the vertex analogue and was introduced by Liu and Deng [2]. Dvořák, Mohar, and Šámal [1] established the currently best upper bound for general graphs. They used a near linear upper bound on the star chromatic index of complete graphs, which states that for every $\varepsilon > 0$ there exists a constant $C$ such that $\chi'_{st}(K_n) \leq C n^{1+\varepsilon}$, for every positive $n$. They asked if the star chromatic index of complete graphs $K_n$ is linear in terms of $n$, which is still an open problem.

The authors in [1] also considered subcubic graphs, proved that for any subcubic graph 7 colors suffice for a star edge-coloring, conjectured that the bound can be decreased to 6, and asked about the bounds for the list version of star edge-coloring. All these questions initiated an intense research on the topic. In the talk, we will give a survey of known results, present the most interesting proofs and techniques, and propose additional possible directions of research on the topic.

References
