A repetition in a sequence is a pair of two identical adjacent blocks. For example, $abab$ is a repetition. A classic theorem by Axel Thue states that there exists an infinite sequence without a repetition. In this talk we present Thue-type results concerning coloring of the Euclidean plane, related to the famous Hadwiger-Nelson problem. Grytczuk, Kosiński and Zmarz [1] introduced an analog of this old problem. Let a sequence of distinct collinear points in $\mathbb{R}^2$ with consecutive distance 1 be called a line path. How many colors are needed for a coloring of $\mathbb{R}^2$ such that the sequence of colors on every line path is nonrepetitive? In [1] a 36-coloring was presented, which was later improved to 18 by Wenus and Węsek [2].

One can observe that actually this 36-coloring satisfy the property even if we relax the distance 1 condition. Let a line $b$-path be a sequence of points on a straight line in $\mathbb{R}^2$ with consecutive distances from the set $[1, b]$. Then in the aforementioned 36-coloring, no line $\sqrt{2}$-path produces a repetition. On the other hand, the 18-coloring does not allow any such relaxation. In our work we prove a certain result in grasshopper pattern avoidance and use it to construct a 30-coloring in which for some $\varepsilon > 0$ no line $(1 + \varepsilon)$-path produces a repetition.

Keywords: nonrepetitive sequence, coloring of the plane, grasshopper pattern avoidance.

AMS Subject Classification: 05C10, 05C15, 68R15.

References
