MONOCHROMATIC MATCHINGS IN GRAPHS WITH AN ORE-TYPE CONDITION

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For given graphs \( G_1, G_2, \ldots, G_c \), the Ramsey number \( R(G_1, G_2, \ldots, G_c) \) is defined as the smallest positive integer \( n \) such that in any \( c \)-coloring of the edges of \( K_n \), there is a monochromatic copy of \( G_i \) in the \( i \)-th color, for some \( i, 1 \leq i \leq c \). There is very little known about \( R(G_1, G_2, \ldots, G_c) \) for \( c \geq 3 \), even for very special graphs. The Ramsey number of matchings was determined by Cockayne and Lorimer [1] and it is proved that 

\[
R(n_1 K_2, n_2 K_2, \ldots, n_c K_2) = n_1 + \sum_{i=1}^{c} (n_i - 1) + 1 \quad \text{for } n_1 \geq n_2 \geq \cdots \geq n_c.
\]

It is a natural question whether a similar conclusion is true if holes are allowed in \( K_{n_1 + \sum_{i=1}^{c} (n_i - 1) + 1} \). The first result in this direction was obtained in [3] where the authors proved that the Ramsey number for two matchings does not change if a graph of maximum degree \( n_1 \) is deleted from \( K_{2n_1 + n_2 - 1} \).

**Theorem 0.1.** [3] For given positive integers \( n_1 \geq n_2 \geq 1 \) and \( t \geq 1 \), 

\[
R(S_t, n_1 K_2, n_2 K_2) = \max\{t, n_1\} + n_1 + n_2 - 1.
\]

In this talk, the Ramsey number of one star versus many matchings is determined which strengthens significantly the result of Gyárfás and Sárközy [3] and also the well-known result of Cockayne and Lorimer.

**Theorem 0.2.** Let \( t \geq 1 \) and \( n_1 \geq n_2 \geq \cdots \geq n_c \) be positive integers. Then 

\[
R(S_t, n_1 K_2, n_2 K_2, \ldots, n_c K_2) = \max\{t, n_1\} + \sum_{i=1}^{c} (n_i - 1) + 1.
\]

As an easy and immediate corollary of Theorem 0.2 we obtain that if \( G \) is a graph obtained from a complete graph on \( R(n_1 K_2, n_2 K_2, \ldots, n_c K_2) \) vertices by deleting the edges of a graph with maximum degree \( (n_1 - 1) \), then \( G \rightarrow (n_1 K_2, n_2 K_2, \ldots, n_c K_2) \). Here, by \( G \rightarrow (n_1 K_2, n_2 K_2, \ldots, n_c K_2) \) we mean if the edges of \( G \) are partitioned into \( c \) disjoint color classes giving \( c \) graphs \( H_1, H_2, \ldots, H_c \), then at least one \( H_i \) has a subgraph isomorphic to \( n_i K_2 \). In this talk we go one step further and consider graphs satisfying an
Ore-type degree condition replacing the minimum degree condition. Here, we call a degree condition **Ore-type** if it gives a lower bound on the degree sum for any two non-adjacent vertices.

**Theorem 0.3.** Assume that \(n_1 \geq n_2 \geq \cdots \geq n_c \geq 1, t \geq 1\) and let \(G\) be a graph on at least \(\max\{t, n_1\} + \sum_{i=1}^{c}(n_i - 1) + 1\) vertices such that for each pair of non-adjacent vertices, the sum of the number of their non-neighbors is at most \(2t - 1\), then \(G \not\rightarrow (n_1K_2, n_2K_2, \ldots, n_cK_2)\).

Theorem 0.3 has some interesting applications which will be discussed. This talk further provides a sharp bound for the maximum number of edges possible in a simple graph \(G\) such that \(G \not\rightarrow (n_1K_2, n_2K_2, \ldots, n_cK_2)\). Uniqueness of these extremal graphs that achieve this edge bound is also established.

**Theorem 0.4.** Let \(n_1 \geq n_2 \geq \cdots \geq n_c\), be positive integers and \(\Lambda = \sum_{i=1}^{c}(n_i - 1)\). If \(G\) is a simple graph with \(n\) vertices, \(n \geq n_1 + \Lambda\) and \(G \not\rightarrow (n_1K_2, n_2K_2, \ldots, n_cK_2)\), then

\[
|E(G)| \leq \max\left\{ \binom{n_1 + \Lambda}{2} + (n - n_1 - \Lambda)(\Lambda - n_1 + 1), \binom{n}{2} - \binom{n - \Lambda}{2}\right\}.
\]

This result provides a far-reaching generalization of an important classical result of Erdős and Gallai and give a new proof for the graph case of the conjecture of Erdős dating back to 1965, known as the Erdős' matching conjecture.

**Keywords:** Matching, Ramsey number, Ore-type condition.

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**References**


