The concept of proper connection number of graphs is an extension of proper colouring and is motivated by rainbow connection of graphs. Andrews et al. [1] and, independently, Borozan et al. [2] introduced the concept of proper connection number as following: A path in an edge-coloured graph $G$ is called a properly coloured path if every two consecutive edges receive distinct colours. The edge-coloured graph $G$ is called properly $k$–connected if every two vertices are connected by at least $k$ internally pairwise vertex-disjoint properly coloured paths. The proper $k$–connection number of $G$, denoted by $pc_k(G)$, is the smallest number of colours that are needed in order to make $G$ properly $k$–connected.

In this talk, we study the proper 2-connection number $pc_2(G)$ of graphs. We prove a new upper for $pc_2(G)$ and determine several classes of graphs satisfying $pc_2(G) = 2$. Among these are all graphs satisfying the Chvátal and Erdős condition ($\alpha(G) \leq \kappa(G)$ with two exceptions). We study the relationship between the proper 2-connection number $pc_2(G)$ and the proper connection number $pc(G)$ of the Cartesian product of two connected graphs.

**Keywords:** properly coloured graph, proper $k$–connection number, Cartesian product.

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**References**
