ON SMALLEST HAMILTONIAN CHAIN SATURATED UNIFORM HYPERGRAPHS

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There are several ways to generalize hamiltonian cycles for k-uniform hypergraphs, $k \geq 3$. We consider such generalization (which is called hamiltonian-chain) introduced in [1], which is probably the strongest one. Namely,

Definition 1 *A cyclic ordering* $(v_1, v_2, \ldots, v_n)$ *of the vertex set of a hypergraph* H *is called a hamiltonian chain if and only if for each* $1 \leq i \leq n$, $\{v_i, v_{i+1}, \ldots, v_{i+k-1}\} =: E_i$ *is an edge of* H.

We say that a hypergraph H is hamiltonian chain saturated if H does not contain a hamiltonian chain but by adding any new edge we create a hamiltonian chain in H. An open problem of G. Y. Katona [2] is to determine the right order of magnitude for the size of smallest k-uniform, $k \geq 3$, hamiltonian chain saturated hypergraphs. We solve this problem by proving that the order is n^{k-1}.

Keywords: saturated hypergraph, hamiltonian cycle, hamiltonian chain.
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References

[1] G.Y. Katona and H.A. Kierstead, Hamiltonian chains in hypergraphs, J. Graph Theory 30 (1999) 205–212.

[2] G.Y. Katona, Hamiltonian chains in hypergraphs, DMTCS proc. AE, 2005, 345–350.