DOMINATION OF SUBGRAPHS

Boštjan Brešar

Faculty of Natural Science and Mathematics,
University of Maribor, Slovenia
e-mail: bostjan.bresar@uni-mb.si

František Kardoš, Ján Katrenič and Gabriel Semanišin

Faculty of Science,
P.J. Šafárik University, Košice, Slovakia
e-mail: frantisek.kardos@upjs.sk, jan.katrenic@upjs.sk, gabriel.semanisin@upjs.sk

We study a kind of domination of sugraphs of prescribed type in a graph G. A subset S of vertices of a graph G is called a *k-path vertex cover* if every path of order k in G contains at least one vertex from S. The minimum cardinality of a k-path vertex cover in G is denoted $\psi_k(G)$.

Such a problem provides a special case of more general Vertex Deletion Problem: Find a minimum weight set of vertices of a given graph whose deletion gives a graph satisfying a given property (see e.g. [4]). Another example of a problem of this type is Feedback Set Problem (see e.g. [2]).

We show that the problem of determining $\psi_k(G)$ is NP-hard for each $k \geq 2$, while for trees the problem can be solved in linear time. We investigate upper bounds on the value of $\psi_k(G)$ and provide several estimations and exact values of $\psi_k(G)$. We also prove that $\psi_3(G) \leq (2n + m)/6$, for every graph G with n vertices and m edges. Moreover, we provide some algorithms for determining the value of ψ_k for some classes of graphs (for more details we refer the reader to [1, 3]).

Keywords: generalised minimum vertex cover, vertex deletion problem, approximation algorithm.

AMS Subject Classification: 05C69, 05C15, 05C85.

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