# H-KERNELS IN INFINITE DIGRAPHS

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Let $H$ be a digraph possibly with loops and $D$ an digraph (possibly infinite) whithout loops whose arcs are coloured with the vertices of $H$ ($D$ is an $H$-coloured digraph). $\mathrm{V}(D)$ and $\mathrm{A}(D)$ will denote the sets of vertices and arcs of $D$ respectively. A directed walk or a directed path $W$ in $D$ is an $H$-walk or an $H$-path if and only if the consecutive colors encountered on $W$ form a directed walk in $H$. A set $N \subseteq \mathrm{V}(D)$ is an $H$-kernel if no two vertices of $N$ have an $H$-path between them and any $u \in \mathrm{V}(D)\backslash N$ reaches some $v \in N$ on an $H$-path. Linek and Sands [4] introduced the concept of $H$-walk and this concept was later used by several authors, in particular, in [2] Galeana-Sánchez and Delgado-Escalante used the concept of $H$-walk in order to introduce the concept of $H$-kernel, which generalizes the concepts of kernel and kernel by monochromatic paths. If $D$ is an infinite digraph, an infinite outward path is an infinite sequence $(v_1, v_2, \ldots)$ of distinct vertices of $D$ such that $(v_i,v_{i+1}) \in A(D)$ for each $i \in \mathbb{N}$.

Let $D$ be an arc-coloured digraph. In [3] Galeana-Sánchez introduced the concept of color-class digraph of $D$, denoted by $\mathscr{C}_C(D)$, as follows: $\mathrm{V}(\mathscr{C}_C(D)) = \{\mathscr{C}_i \mid \mathscr{C}_i$ is the subdigraph of $D$ whose arcs are the arcs of $D$ coloured $i\}$ and $(\mathscr{C}_i,\mathscr{C}_j) \in A(\mathscr{C}_C(D))$ if and only if there exist two arcs namely $f = (u,v) \in A(D)$ coloured $i$ and $g = (v,w) \in A(D)$ coloured $j$. Since $\mathrm{V}(\mathscr{C}_C(D)) \subseteq \mathrm{V}(H)$, the main question is: What structural properties of $\mathscr{C}_C(D)$, respect to $H$, imply that $D$ has an $H$-kernel? Suppose that $D$ has no infinite outward $H$-path. In this talk we are going to see that if $\mathscr{C}_C(D) \subseteq H$, then $D$ has an $H$-kernel. We will see that if there exists a partition $(V_1,V_2)$ of $\mathrm{V}(\mathscr{C}_C(D))$ such that: (1) $\mathscr{C}_C(D)[V_i] \subseteq H[V_i]$ for each $i \in \{1,2\}$ and (2) if $(u,v) \in A(\mathscr{C}_C(D))$ for some $u \in V_i$ and for some $v \in V_j$, with $i \neq j$ and $i,j \in \{1,2\}$, then $(u,v) \notin A(H)$. Then $D$ has an $H$-kernel.

**Keywords:** Kernel, Kernel by monochromatic paths, $H$-kernel, Color-class digraph.

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## References

[1] P. Arpin, V. Linek, Reachability problems in edge-colored digraphs. Discrete Math. 307 (2007) 2276-2289.

[2] P. Delgado-Escalante, H. Galeana-Sánchez, Restricted domination in arc-colored digraphs. Disc. Appl. Math., In press.

[3] H. Galeana-Sánchez, Kernels by monochromatic paths and the color-class digraph, Discuss. Math. Graph Theory, In press.

[4] V. Linek, B. Sands, Anote on paths in edge-colored tournaments, Ars Combin. 44 (1996) 225-228.