ON (I, J, K)-COLORING OF PLANAR GRAPHS

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Let $\mathcal{F}$ be the family of planar graphs without cycle of length 4 and 5. Steinberg's Conjecture (1976) [2] that says every graph of $\mathcal{F}$ is 3-colorable remains widely open. Focusing on a relaxation proposed by Erdős (1991), many studies proved the conjecture for some subfamilies of $\mathcal{F}$. For example, Borodin *et al.* [1] proved that every planar graph without cycles of length 4 to 7 is 3-colorable.
A graph $G = (V, E)$ is said to be (i, j, k)-colorable if its vertex set can be partitioned into three sets V_1, V_2, V_3 such that the graphs $G[V_1], G[V_2]$, and $G[V_3]$ induced by the vertices of V_1, V_2, V_3 have maximum degree at most i, j, k respectively. Under this terminology, Steinberg's Conjecture says that every graph of $\mathcal{F}$ is $(0, 0, 0)$-colorable. A result of Xu [3, 4] implies that every graph of $\mathcal{F}$ is $(1, 1, 1)$-colorable. We will give obtained results for (i, j, k)-colorings of graphs of $\mathcal{F}$.

Keywords: graph, coloring, .

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