ON THE CROSSING NUMBER OF PRODUCTS OF 6-VERTEX GRAPHS WITH PATHS

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Let G_1 and G_2 be simple graphs with vertex sets $V(G_1)$ and $V(G_2)$, and edge sets $E(G_1)$ and $E(G_2)$. The Cartesian product $G_1 \times G_2$ of graphs G_1 and G_2 has vertex set $V(G_1 \times G_2) = V(G_1) \times V(G_2)$ and any two vertices (u, u') and (v, v') are adjacent in $G_1 \times G_2$ if and only if either $u = v$ and u' is adjacent with v' in G_2, or $u' = v'$ and u is adjacent with v in G_1. The crossing number, $cr(G)$, of a graph G is the minimal number of pairwise intersections of nonadjacent edges in any drawing of G in the plane.

There are known exact values concerning the crossing numbers of the Cartesian product of paths, cycles and stars with some graphs of order four and five (see [1], [3], [4]). The similar problem was solved in [2] also for the Cartesian product of cycles with 6-vertex trees. The aim of this presentation is to determine the crossing numbers of the Cartesian product of the path P_n on n edges with some graphs of order six. We give first the crossing number of the Cartesian product of the special graph H on six vertices with the path and then we use this result to find crossing numbers of Cartesian product of another 6-vertex graphs with the path P_n.

Keywords: graph, drawing, crossing number, Cartesian product.

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