

SEARCHING FOR SNARKS AMONGST CAYLEY GRAPHS

DRAGAN MARUŠIČ

University of Primorska and University of Ljubljana

e-mail: dragan.marusic@famnit.upr.si

KLAVDIJA KUTNAR

University of Primorska

e-mail: klavdija.kutnar@upr.si

A *snark* is a connected, bridgeless cubic graph with chromatic index equal to 4. In [1] it was conjectured that there are no snarks amongst Cayley graphs. (A *Cayley graph* on a group G relative to a subset $S = S^{-1} \subseteq G \setminus \{1\}$ has vertex set G and edges of the form $\{g, gs\}$, $g \in G$, $s \in S$.) It follows from [2, 4] that giving an answer to the above conjecture essentially boils down to determining whether there are snarks amongst Cayley graphs on simple groups.

We will present an innovative approach to finding a possible solution to the above conjecture combining the theory of Cayley maps with a classical result about maximum dominating forests in cyclically 4-edge connected cubic graphs [3]. Partial results obtained with this approach will be discussed in detail.

Keywords: snark, cubic graph, coloring, Cayley graph.

AMS Subject Classification: 05C25, 05C15, 20E32.

References

- [1] B. Alspach, Y.-P. Liu, and C.-Q. Zhang, Nowhere-zero 4-flows and Cayley graphs on solvable groups, *SIAM J. Discrete Math.* **9** (1996), 151–154.
- [2] R. Nedela and M. Škovič, Cayley snarks and almost simple groups, *Combinatorica* **21** (2001), 583–590.
- [3] C. Payan and M. Sakarovitch, Ensembles cycliquement et graphes cubiques, *Cahiers du Centre D'études de Recherche Operationelle* **17** (1975), 319–343.

- [4] P. Potočnik, Edge-colourings of cubic graphs admitting a solvable vertex-transitive group of automorphisms, *J. Combin. Theory Ser. B* **91** (2004), 289-300.