Strategic balance in graphs

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Abstract. For a graph $G = (V, E)$, a nonempty subset S contained in V is called an *alliance* if for each $v \in S$, there are at least as many vertices from the closed neighborhood of v in S as in $V \setminus S$. An alliance is called *global* if it is also a dominating set of G. The *alliance partition number* of G was defined in [1] to be the maximum number of sets in a partition of $V(G)$ such that each set is an alliance. Similarly, in [3] the *global alliance partition number* is defined for global alliances, where the authors studied the problem for (binary) trees.
In this talk we define a problem of *strategic balance*: given a graph G determine whether there is a partition of vertex set $V(G)$ into three subsets N, S and B such that both N and S are global alliances. We give a survey of its general properties, e.g., showing that graph G has a strategic balance iff its global alliance partition number is at least 2. We constructed a polynomial time algorithm for solving the problem for trees (thus giving an answer to the open question stated in [3]) and studied this problem for many classes of graphs: cycles, wheels, stars, complete graphs, and complete k-partite graphs. Moreover, we proved that this problem is NP-complete for graphs with degree bounded by 4.

Keywords: global alliance, alliance partitioning number, strategic balance

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