MINIMIZING JUMPS OF INTERVAL ORDERS THROUGH GRAPH COVERING

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We investigate the problem of determining the jump number of a poset, which is NP-complete, even in the restricted case of interval orders [3]. Jump number is the minimum number of noncomparable adjacent elements in a linear extension of a given ordered set. The monograph [1] contains comprehensive background material on interval orders, i.e., posets realizable in form of intervals on the real line.

An appealing algorithm for the jump number problem on interval posets appeared in [3] (see also [2]). In this approach, the problem is reduced to packing vertex-disjoint edges and odd cycles in a graph based on the canonical representation of an interval order. We establish in a sense a complementary result: a reduction to covering problem, where a graph is to be covered by the minimum-weight collection of edges and odd cycles (not necessarily disjoint) from a given family. The proof is of algorithmic nature and may be used to generate jump-optimal or nearly-optimal linear extensions by invoking the algorithms for particular cases of graph or set covering problems. In addition, since the NP-hardness proof involves a reduction from exact cover by 3-sets, our result may be seen to supplement this fact with a transformation in reverse direction.

Keywords: poset, interval order, jump number, graph packing, set cover.
AMS Subject Classification: 90C27.

References

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