Domination game

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In this talk a domination game (introduced in B. Brešar, S. Klavžar, D.F. Rall, Domination game and an imagination strategy, SIAM J. Discrete Math. 24 (2010) 979–991) will be presented. Just like in the classical coloring game variant, we have a player, called Dominator, who wishes to dominate a graph in as few steps as possible and another player, called Staller, who wishes to delay the process as much as possible. The players alternate their moves and vertices must be chosen in such a way that whenever a vertex is chosen by either player, at least one additional vertex of the graph G is dominated that was not dominated by the vertices previously chosen. There are two different games, in Game 1 Dominator starts the game while in Game 2 Staller has the first move. The *game domination number* $\gamma_g(G)$ of a graph G is the total number of vertices chosen when Game 1 is played on G using optimal strategies by both player. Similarly, the *Staller-start game domination number* $\gamma'_g(G)$ of G, is the cardinality of the set of vertices chosen when Game 2 is played on G.

An overview of what is know about this domination game will be given. In particular, $\gamma_g(G)$ and $\gamma'_g(G)$ can differ by at most one (proved in part in a manuscript by Bill Kinnersley and his co-workers), and most of possible pairs $(\gamma_g(G), \gamma'_g(G))$ can be realized on trees. Bounds on $\gamma_g(G)$ will be given for general graphs and for particular classes of graphs. The domination game will also be connected with Vizing's conjecture. Numerous open problems and conjectures will also be stated.