

INTERVAL EDGE-COLORINGS OF CARTESIAN PRODUCTS OF GRAPHS

PETROS PETROSYAN, HRANT KHACHATRIAN

*Department of Informatics and Applied Mathematics,
Yerevan State University*

e-mail: pet_petros@ipia.sci.am, hrant@egern.net

AND HOVHANNES TANANYAN

*Department of Applied Mathematics and Informatics,
Russian-Armenian State University*

e-mail: HTananyan@yahoo.com

An edge-coloring of a graph G with colors $1, 2, \dots, t$ is called an interval t -coloring [1] if all colors are used and the colors of edges incident to each vertex of G are distinct and form an interval of integers. Let $\mathfrak{N}$ be the set of all interval colorable graphs. For a graph $G \in \mathfrak{N}$, the greatest value of t for which G has an interval t -coloring is denoted by $W(G)$. In this paper we prove that if $G, H \in \mathfrak{N}$ and H is an r -regular graph, then $G \square H \in \mathfrak{N}$ and $W(G \square H) \geq W(G) + W(H) + r$. We also give some sufficient conditions for $W(G \square H) \geq W(G) + W(H) + \text{diam}(G) \cdot r$ when $G, H \in \mathfrak{N}$ and H is an r -regular graph. In particular, we show that if G is an r -regular graph and $G \in \mathfrak{N}$, then $W(G \square P_m) \geq W(G) + W(P_m) + (m - 1)r$ ($m \in \mathbf{N}$) and $W(G \square C_{2n}) \geq W(G) + W(C_{2n}) + nr$ ($n \geq 2$). Next, we consider n -dimensional toroidal grids and Hamming graphs and provide some partial results for these graphs. Finally, we confirm the conjecture on the n -dimensional cube Q_n [2] and show that Q_n has an interval t -coloring if and only if $n \leq t \leq \frac{n(n+1)}{2}$.

Keywords: edge-coloring, interval coloring, regular graph, Hamming graph.

AMS Subject Classification: 05C15, 05C76.

References

- [1] A.S. Asratian, R.R. Kamalian, Interval colorings of edges of a multi-graph, Appl. Math. 5 (1987) 25–34 (in Russian).
- [2] P.A. Petrosyan, Interval edge-colorings of complete graphs and n -dimensional cubes, Discrete Mathematics 310 (2010) 1580–1587.