TRANSVERSALS IN MINIMUM DOMINATING SETS FOR SPLIT GRAPHS

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We consider a finite set V and an optimization problem on V: the search for a maximum (or minimum) cardinality subset of V verifying a given property P. We search for vertices contained in all or in no minimum dominating set especially in the case of split graphs. Most of similar studies interested in dominating sets consider trees. By forced vertices, we mean those contained in every minimum dominating set whereas excluded vertices do not belong to any of them. The d-transversals and the d-blockers are more general concepts. A d-transversal is a subset of V which intercepts any optimum solution in at least d elements while a d-blocker is a subset of V whole removal deteriorates the value of an optimum solution by at least d. We review d-transversals and d-blockers for some situations studied in the literature (matchings, stable sets), we give necessary and sufficient conditions for excluded and forced vertices in general graphs. It appears that unlike stable sets a 1-transversal in dominating sets is not necessary a 1-blocker. For split graphs, we give some sufficient conditions to define excluded and forced vertices and we propose partitioning the split graph into disjoint split subgraphs in order to deduce forced vertices. So, one could approach a minimum dominating set by branching over forced vertices. One could also try to find bounds of a minimum d-transversal (resp. d-blocker) cardinality for any fixed $d \geq 1$.

Keywords: dominating sets, transversals, split graphs.

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