ADDITIVITY AND FACTORIZABILITY OF INDUCED HEREDITARY GRAPH PROPERTIES

Ewa Drgas-Burchardt

Faculty of Mathematics Computer Science and Econometrics
University of Zielona Góra
e-mail: E.Drgas-Burchardt@wmie.uz.zgora.pl

A graph property is any class of graphs that is closed under isomorphisms. A graph property is induced hereditary if it is closed under taking induced subgraphs.
Let $\mathcal{P}_1, \ldots, \mathcal{P}_n$ be induced hereditary graph properties and H be a graph with $V(H) = \{v_1, \ldots, v_n\}$. The symbol $H[\mathcal{P}_1, \ldots, \mathcal{P}_n]$ denotes the class of all graphs G for which there exists a partition $(V_1, \ldots, V_n)$ of $V(G)$ such that for each $i \in \{1, \ldots, n\}$ the condition $G[V_i] \in \mathcal{P}_i$ holds, and if $xy \in E(G)$ with $x \in V_i$ and $y \in V_j$, $i \neq j$, then $v_iv_j \in E(H)$. Such a partition is called $H[\mathcal{P}_1, \ldots, \mathcal{P}_n]$-partition of G.

Let H be a graph. A graph property $\mathcal{P}$ is properly H-factorizable over the class of all induced hereditary graph properties if there exist induced hereditary graph properties $\mathcal{P}_1, \ldots, \mathcal{P}_n$ such that:
(1) $\mathcal{P} = H[\mathcal{P}_1, \ldots, \mathcal{P}_n]$ for some ordering of $V(H)$, and
(2) there is a graph G^* such that for each $H[\mathcal{P}_1, \ldots, \mathcal{P}_n]$-partition $(V_1 \ldots, V_n)$ of G^* the condition $V_i \neq \emptyset$ holds, $i \in \{1, \ldots, n\}$, and for each $v_iv_j \in E(H)$ there exists $xy \in E(G^*)$ satisfying $x \in V_i$ and $y \in V_j$.

For given graphs $G_1, \ldots, G_n$ and a graph H with $V(H) = \{v_1, \ldots, v_n\}$ we will use the symbol $H[G_1, \ldots, G_n]$ to denote the graph whose vertex set is the union of $V(G_1), V(G_2), \ldots, V(G_n)$ and whose edge set consists of the union of $E(G_1), E(G_2), \ldots, E(G_n)$ with the additional edge set $\{xy : x \in V(G_i),\ y \in V(G_j),\ v_iv_j \in E(H)\}$.

Let H be a graph. An induced hereditary graph propery $\mathcal{P}$ is H-additive if a condition $G_1, \ldots, G_n \in \mathcal{P}$ yields $H[G_1, \ldots, G_n] \in \mathcal{P}$ for any ordering of $V(H)$.

In this talk we analyze relationship between graphs A, B, C such that given induced hereditary graph property $\mathcal{P}$ is properly A-factorizable, B-additive and for which C is a minimal forbidden graph.

Keywords: graph property, minimal forbidden graph, reducibility, additivity.

AMS Subject Classification: 05C75, 05C15.