ADDITIVITY AND FACTORIZABILITY OF INDUCED HEREDITARY GRAPH PROPERTIES

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A graph property is any class of graphs that is closed under isomorphisms. A graph property is induced hereditary if it is closed under taking induced subgraphs.

Let $P_1, \ldots, P_n$ be induced hereditary graph properties and $H$ be a graph with $V(H) = \{v_1, \ldots, v_n\}$. The symbol $H[P_1, \ldots, P_n]$ denotes the class of all graphs $G$ for which there exists a partition $(V_1, \ldots, V_n)$ of $V(G)$ such that for each $i \in \{1, \ldots, n\}$ the condition $G[V_i] \in P_i$ holds, and if $xy \in E(G)$ with $x \in V_i$ and $y \in V_j$, $i \neq j$, then $v_iv_j \in E(H)$. Such a partition is called $H[P_1, \ldots, P_n]$-partition of $G$.

Let $H$ be a graph. A graph property $P$ is properly $H$-factorizable over the class of all induced hereditary graph properties if there exist induced hereditary graph properties $P_1, \ldots, P_n$ such that:

1. $P = H[P_1, \ldots, P_n]$ for some ordering of $V(H)$, and
2. there is a graph $G^*$ such that for each $H[P_1, \ldots, P_n]$-partition $(V_1, \ldots, V_n)$ of $G^*$ the condition $V_i \neq \emptyset$ holds, $i \in \{1, \ldots, n\}$, and for each $v_iv_j \in E(H)$ there exists $xy \in E(G^*)$ satisfying $x \in V_i$ and $y \in V_j$.

For given graphs $G_1, \ldots, G_n$ and a graph $H$ with $V(H) = \{v_1, \ldots, v_n\}$ we will use the symbol $H[G_1, \ldots, G_n]$ to denote the graph whose vertex set is the union of $V(G_1), V(G_2), \ldots, V(G_n)$ and whose edge set consists of the union of $E(G_1), E(G_2), \ldots, E(G_n)$ with the additional edge set $\{xy : x \in V(G_i), \ y \in V(G_j), \ v_iv_j \in E(H)\}$.

Let $H$ be a graph. An induced hereditary graph property $P$ is $H$-additive if a condition $G_1, \ldots, G_n \in P$ yields $H[G_1, \ldots, G_n] \in P$ for any ordering of $V(H)$.

In this talk we analyze relationship between graphs $A$, $B$, $C$ such that given induced hereditary graph property $P$ is properly $A$-factorizable, $B$-additive and for which $C$ is a minimal forbidden graph.

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