EQUALITY OF DOMINATION AND TRANSVERSAL NUMBERS IN HYPERGRAPHS

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The domination number $\gamma(\mathcal{H})$ and the transversal number $\tau(\mathcal{H})$ (also called vertex covering number) of a hypergraph $\mathcal{H}$ are defined analogously to those of a graph. A hypergraph is of rank k if each edge contains at most k vertices. The inequality $\tau(\mathcal{H}) \geq \gamma(\mathcal{H})$ is valid for every hypergraph $\mathcal{H}$ without isolated vertices. We study the structure of hypergraphs satisfying $\tau(\mathcal{H}) = \gamma(\mathcal{H})$, moreover prove that the corresponding recognition problem is NP-hard already on the class of linear hypergraphs of rank 3. We focus our attention mostly on hypergraphs for which $\tau = \gamma$ hereditarily holds, that is in which each subhypergraph $\mathcal{H}'$ without isolated vertices fulfills the equality $\tau(\mathcal{H}') = \gamma(\mathcal{H}')$. We prove that if each induced subhypergraph satisfies the equality then it holds for the non-induced ones as well. Moreover, for every positive integer k, there are only a finite number of forbidden subhypergraphs of rank k, and each of them has domination number at most k. Thus, hypergraphs for which $\tau = \gamma$ hereditarily holds can be recognized in polynomial time if the rank is fixed.

Keywords: hypergraph, domination number, transversal number.

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