COUNTABLE UNIVERSAL GRAPHS FOR PROPERTIES OF COUNTABLE GRAPHS

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Rado constructed a (simple) denumerable graph R with the positive integers as vertex set with the following edges: For given m and n with $m < n$, m is adjacent to n if n has a 1 in the m'th position of its binary expansion. It is well known that R is a universal graph in the set $\mathcal{I}_c$ of all countable graphs since every graph in $\mathcal{I}_c$ is isomorphic to an induced subgraph of R. We prove constructively the existence of an abundance of copies of R in R.

The existence of a countable universal graph for any induced-hereditary property of countable graphs is guaranteed by a general recursive construction; a general construction of universal graphs for products of properties of graphs will also be discussed.

Graphs which are universal in or for $\mathcal{P}$ for different induced-hereditary properties $\mathcal{P}$ of countable graphs can sometimes be obtained by restricting the edges of R. Constructions of such universal graphs are discussed for the set of countable graphs with colouring-number at most $k + 1$ and k-degenerate graphs.

Let H be any countable graph. A construction of a graph $U(H)$ which is H-universal in $\to H_c$, the hom-property of countable H-colourable graphs, is also discussed and characterised. The graphs H for which $U(H) \cong R$ are also characterised.

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