KERNELS BY MONOCHROMATICS PATHS IN DIGRAPHS WHERE ONE OF ITS SUBDIGRAPHS HAS NO γ-CYCLES .

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We call the digraph D an m-colored digraph if its arcs are colored with m colors. If D is an m-colored digraph and $a \in A(D)$, $color(a)$ will denote the color has been used in a. A path (or a cycle) is called monochromatic if all of its arcs are colored alike. A cycle is called a quasi-monochromatic cycle if with at most one exception all of its arcs are colored alike. A γ-cycle in D is a sequence of different vertices, $\gamma = (u_0, u_1, ..., u_n, u_0)$ such that for every $i \in \{0, 1, ..., n\}$ there exists a $u_i u_{i+1}$-monochromatic path in D and there is no $u_{i+1} u_i$-monochromatic path in D. A digraph D is called transitive by monochromatic paths if the existence of a xy-monochromatic path in D and a yz-monochromatic path in D imply that there is a xz-monochromatic path in D. A set $N \subseteq V(D)$ is independent and dominant by monochromatic paths (kernel by monochromatic paths) if it satisfies the following two conditions: (i) for every pair of different vertices $u, v \in N$ there is no monochromatic path between them and; (ii) for every vertex $x \in V(D) \setminus N$ there is a vertex $y \in N$ such that there is an xy-monochromatic path. The closure of D, denoted by $\mathscr{C}(D)$ is the m-colored multidigraph defined as follows: $V(\mathscr{C}(D)) = V(D)$, $A(\mathscr{C}(D)) = A(D) \cup \{(u, v)$ with color i: there exists an uv-monochromatic path colored i contained in $D\}$. Notice that for any digraph D $\mathscr{C}(\mathscr{C}(D)) \cong \mathscr{C}(D)$ and D has a kernel by monochromatic paths if and only if $\mathscr{C}(D)$ has a kernel. We say that $C_3 = (u_0, u_1, u_2, u_0)$ (the directed cycle of length 3) is a 3-colored $C_3 - (C_1, C_1, C_2)$ if $a = color((u_0, u_1)) \in C_1$, $b = color((u_1, u_2)) \in C_1$ and $c = color((u_2, u_0)) \in C_2$ with $a \neq b$, $b \neq c$, and $a \neq c$. We say that $P_3 = (u_0, u_1, u_2, u_3)$ (the directed path of length 3) is a 3-colored $P_3 - (C_1, C_1, C_2)$ if $a = color((u_0, u_1)) \in C_1$, $b = color((u_1, u_2)) \in C_1$ and $c = color((u_2, u_0)) \in C_2$ with $a \neq b$, $b \neq c$, and $a \neq c$.

Let D be a finite m-colored digraph. Suppose that there is a partition $C = C_1 \cup C_2$ of the set of colors of D such that $D_1 = D[\{a \in F(D) : color(a) \in C_1\}]$ has no γ-cycles and $D_2 = D[\{a \in F(D) : color(a) \in C_2\}]$ is transitive by monochromatic paths. In this talk we will discuss the following result: if D is a digraph with the structure mentioned above and $\mathscr{C}(D)$ meets the following two conditions: (i) all 3-colored $C_3 - (C_1, C_1, C_2)$ has at least two symmetrical arcs and; (ii) if (u, v, w, x) is a 3-colored $P_3 - (C_1, C_1, C_2)$ then $(u, x) \in A(\mathscr{C}(D))$. Then D has a kernel by monochromatic paths.

This result is a wide extension of the original result by B. Sands, N. Sauer and R. Woodrow that asserts: Every 2-colored digraph has a kernel by monochromatic paths.

Keywords: digraph, kernel, kernel by monochromatic paths, γ-cycle.

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