**13th WORKSHOP ON GRAPH THEORY**

# COLOURINGS, INDEPENDENCE AND DOMINATION

# CID

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# ABSTRACTS

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## Abstracts

# THE ORDER OF HYPOTRACEABLE ORIENTED GRAPHS

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An oriented graph $D$ is *traceable* if it contains a path that visits every vertex and is *hypotraceable* if $D$ is not traceable but $D - v$ is traceable for every $v \in V(D)$. Grötschel *et al.* constructed in [1] and [2] an infinite family of hypotraceable oriented graphs, the smallest of which has order 13. These are the only constructions of hypotraceable oriented graphs that appear in the literature. We show that there exist hypotraceable oriented graphs of order $n$ for every $n \geq 8$, except possibly for $n = 9, 11$.

**Keywords:** hypotraceable; hypohamiltonian; $k$-traceable; oriented graph.

**AMS Subject Classification:** 05C20, 05C38.

## REFERENCES

[1] M. Grötschel, C. Thomassen and Y. Wakabayashi, *Hypotraceable digraphs*, J. Graph Theory **4** (1980) 377–381.

[2] Grötschel and Y. Wakabayashi, Constructions of Hypotraceable Digraphs, Mathematical Programming, R.W. Cottle, M.L. Kelmanson and B. Korte (Eds), Elsevier Science Publishers B.V. (North-Holland, 1984) 147–175.

# SOME PROPERTIES OF LOCAL KNESER GRAPHS

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*Local Kneser graph* $U_t(n, r)$ is defined in [3]. The graph $U_t(n, r)$ is a generalization of local complete graph $U(n, r)$ ([2]) and defined as follows.

Let $n$, $r$ and $t$ be positive integers where $n \geq r \geq 2t$. Set $U_t(n, r)$ to be the local Kneser graph whose vertex set contains all ordered pairs $(A, B)$ such that $|A| = t$, $|B| = r - t$, $A, B \subseteq [n]$ and $A \cap B = \varnothing$. Also, two vertices $(A, B)$ and $(C, D)$ of $U_t(n, r)$ are adjacent if $A \subseteq D$ and $C \subseteq B$.

In this talk, we review some properties of local Kneser graphs [1]. In this regard, as a generalization of the Erdös-Ko-Rado theorem, we characterize the maximum independent sets of local Kneser graphs. Next, we provide an upper bound for their chromatic number.

**Keywords:** Erdös-Ko-Rado theorem, graph homomorphism, local chromatic number.

**AMS Subject Classification:** 05C.

## REFERENCES

[1] Meysam Alishahi, Hossein Hajiabolhassan, Ali Taherkhani, A Generalization of the Erdös-Ko-Rado Theorem, manuscript, 2009.

[2] P. Erdős, Z.Füredi, A. Hajnal, P. Komjáth, V. Rödl and Á. Seress, Coloring graphs with locally few colors, Discrete Math. **59** (1986) 21–34.

[3] János Körner, Concetta Pilotto and Gábor Simonyi, Local chromatic number and sperner capacity, J. Combin. Theory (B) **95** (2005) 101–117.

This paper is partially supported by center of excellence in algebraic and logical structure in discrete mathematics in Shahid Beheshti University.

## RANDOM PROCEDURES FOR DOMINATING SETS IN BIPARTITE GRAPHS

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Using multilinear functions and random procedures, new upper bounds on the domination number of a bipartite graph in terms of the cardinalities and the minimum degrees of the two color classes are established.

# THE EQUAL SUM FREE SUBSET PROBLEM

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In practice, competitions can occur, where the competitors have weights and we want to compare two groups with equal sum of weights. One of the theoretical problems involved here is the following:

Let $w_1, w_2, \ldots, w_n$ be a sequence of positive integers (repetition is allowed). A set $I \subseteq \{1, \ldots, n\}$ is dependent if there exist nonempty subsets $J, K \subseteq I$ such that

1) The sum of the for $j \in J$ is equal to the sum for $k \in K$
2) The sets $\{w_j : j \in J\}$ and $\{w_k : k \in K\}$ are disjoint.

A set $I$ is independent if it is not dependent. Given $n$, the question is, how large independent set we can guarantee for any sequence of length $n$.

# CHROMATIC AND FLOW UNIQUENESS IN A FAMILY OF 2-CONNECTED GRAPHS

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Chromatic and flow polynomials of a graph are two important cases of its Tutte polynomial. Each of them contains much information of the graph. We present some infinite classes of 2-connected graphs determined by their chromatic polynomials and flow polynomials together, but which are not chromatic unique and are not flow unique. Some interesting results in this subject are published in [1] (see the references in [1] as well).

**Keywords:** chromatic polynomial, chromatic uniqueness, flow polynomial, flow uniqueness, chromatic and flow uniqueness.

**AMS Subject Classification:** 05C15.

## REFERENCES

[1] Yinghua Duan, Haidong Wu, Qinglin Yu, On chromatic and flow polynomial unique graphs, Discrete Appl. Math. **156** (2008) 2300–2309.

# AN UPPER BOUND FOR THE SIZE OF A SMALLEST INDEPENDENT DOMINATING SET OF A GRAPH

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Let $G$ be a connected graph with $|V(G)| \geq 2$ (where $V(G)$ is the vertex set of $G$). Let $d(G)$ denote the maximum vertex degree in $G$ and let $\mu(G)$ denote the size of a smallest independent dominating set of $G$. Two trivial upper bounds for $\mu(G)$ are $|V(G)| - d(G)$ and $\frac{d(G)}{d(G)+1}|V(G)|$. We introduce a new parameter $d'(G)$ that is at most equal to $d(G)$, and we present the following improvement of the latter bound, assuming $G$ is not complete:

$$\mu(G) \leq \frac{d'(G)-1}{d'(G)}|V(G)|.$$

We exhibit graphs $G$ for which

$$\mu(G) = |V(G)| - d(G) = \frac{d'(G)-1}{d'(G)}|V(G)| - d'(G) + 2,$$

and we conjecture that the right-hand side of the second equality is an upper bound for any connected graph $G$.

**Keywords:** independent set, dominating set.

**AMS Subject Classification:** 05C35, 05C69.

# ON-LINE RANKING OF SPLIT-GRAPHS

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A vertex ranking of a simple graph $G$ is an assignment of integers to the vertices of $G$ such that each path connecting two vertices of the same color contains a vertex of a bigger color. The goal is to find a vertex ranking using as few colors as possible. In this paper we consider the on-line setting of the vertex ranking problem for split graphs. We prove that the worst case ratio of the number of colors used by any on-line ranking algorithm and the number of colors used in optimal off-line solution may be arbitrarily large. This negative result gives motivation to consider the semi on-line problem, where the split graph is presented on-line but its clique number is given in advance. We also prove that any semi on-line ranking algorithm may be forced to use $2\chi_r(G)$ colors, where $\chi_r(G)$ is the (off-line) vertex ranking number of $G$. Finally, the semi on-line algorithm which achieves this bound, i.e. uses $2\chi_r(G)$ colors in the worst case is given.

## References

[1] P. Borowiecki, On-line coloring of graphs, in: M. Kubale (ed.): *Graph Colorings*, *Contemporary Mathematics* 352, American Mathematical Society (2004) 21-33

[2] E. Bruoth, M. Horňák, On-line ranking number for cycles and paths, *Discussiones Mathematicae Graph Theory* 19 (1999) 175-197

[3] E. Bruoth, M. Horňák, A lower bound for on-line ranking number of a path, *Discrete Mathematics* 307(11-12) (2007) 1347-1355

[4] D. Dereniowski, Rank coloring of graphs, in: M. Kubale (ed.): *Graph Colorings*, *Contemporary Mathematics* 352, American Mathematical Society (2004) 79-93

[5] P.L. Hammer, B. Simeone, The splittance of a graph, *Combinatorica* 1(3) (1981) 275-284

[6] G. Isaak, R. Jamison, D. Narayan, Greedy rankings and arank numbers, *Information Processing Letters* (2009) (to appear)

[7] M. Kubale, *Graph Colorings*, *Contemporary Mathematics* 352, American Mathematical Society (2004)

[8] I. Schiermeyer, Zs. Tuza, M. Voigt, On-line rankings of graphs, *Discrete Mathematics* 212(1-2) (2000) 141-147

[9] G. Semanišin, R. Soták, A note on on-line ranking number of graphs, *Czechoslovak Mathematical Journal* 56(2) (2006) 591-599

# ON INDEPENDENT VERTEX SETS AND INDUCED MATCHINGS

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(joint work with Chính T. Hoáng, Van Bang Le, Vadim V. Lozin, Raffaele Mosca and Ragnar Nevries)

Let $G = (V, E)$ be a finite undirected graph. A vertex set $S \subseteq V$ is *independent* (or *stable*) if the vertices in $S$ are mutually nonadjacent. For given $G$, the MAXIMUM INDEPENDENT SET (MIS) Problem asks for an independent vertex set of maximum size in $G$. We discuss various techniques for solving the MIS problem efficiently on particular graph classes, and mention some open problems. It is well known that clique separator decomposition and modular decomposition are helpful tools for solving the MIS problem. One of our results allows to combine both of them. This implies various improvements of known results, among them a polynomial time algorithm for MIS on the class of apple-free graphs which is a common generalization of chordal graphs as well as of claw-free graphs.

For given $G$, the MAXIMUM INDUCED MATCHING (MIM) Problem asks for an independent node set in the square $L(G)^2$ of the line graph $L(G)$ of $G$, i.e., the nodes of $L(G)^2$ are the edges of $G$, and two distinct edges are adjacent in $L(G)^2$ if they intersect each other or see each other in $G$. It is well known that, unlike the Maximum Matching Problem, MIM is NP-complete, and it remains NP-complete even for (very restricted subclasses of) bipartite graphs and for line graphs. On the other hand, the problem is efficiently solvable for a variety of graph classes such as (weakly) chordal graphs and graphs of bounded clique-width; many papers are dealing with the complexity of the MIM problem on particular graph classes. We discuss the complexity of this problem and its generalization in $L(G)^k$ for $k \geq 3$ for some important graph classes such as chordal graphs and strongly chordal graphs.

For given $G$, the DOMINATING INDUCED MATCHING (DIM) Problem (also called EFFICIENT EDGE DOMINATION (EED) Problem in various papers) asks for the existence of an independent node set in $L(G)^2$ which simultaneously is a dominating set in $L(G)$. It is well known that the DIM problem is NP-complete even on (very restricted subclasses of) bipartite graphs, and its complexity has been open for weakly chordal graphs and even for chordal bipartite graphs. One of our results shows that this problem is solvable in polynomial time for hole-free graphs.

## 3-CONSECUTIVE C-COLORINGS OF GRAPHS *

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A 3-consecutive C-coloring (3CC-coloring) of a graph $G = (V, E)$ is a mapping $\varphi : V \to \mathbb{N}$ such that no three-colored path $P_3$ occurs. That is, among any three consecutive vertices there exist at least two having the same color. This coloring constraint has equivalent descriptions in the theory of mixed [3] and color-bounded [1] hypergraphs. Moreover, it is closely related to the 3-consecutive colorings of graphs [2].

The maximum number $\bar{\chi}_{3CC}(G)$ of colors, that can occur in a 3CC-coloring of a graph $G$, is the 3-consecutive upper chromatic number of $G$. In this talk we present general estimates on $\bar{\chi}_{3CC}(G)$ in terms of several graph parameters. In particular, exactly determined values of $\bar{\chi}_{3CC}(G)$ will be given if $G$ is a tree or a unicyclic graph.

On the other hand, we characterize graphs admitting 3-consecutive C-colorings with exactly three and exactly four colors, respectively.

**Keywords:** graph coloring, vertex coloring, consecutive coloring, upper chromatic number.

**AMS Subject Classification:** 05C15.

### REFERENCES

[1] Cs. Bujtás and Zs. Tuza, Color-bounded hypergraphs, I: General results, Discrete Mathematics, (2009), doi: 10.1016/j.disc.2008.04.019, in print.

[2] E. Sampathkumar, M.S. Subramanya and Charles Dominic, 3-Consecutive Vertex Coloring of a Graph, Proceedings of the International Conference on Discrete Mathematics, (2008), 147–151.

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[3] V.I. Voloshin, Coloring Mixed Hypergraphs – Theory, Algorithms and Applications, Fields Institute Monographs 17, AMS, 2002.

## ON DOMINATION IN GRAPH PRODUCTS

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The well-known conjecture on domination number due to Vizing (1963) says that for two graphs $G$ and $H$ and their cartesian product $G\Box H$, $\gamma(G)\cdot\gamma(H) \leq \gamma(G\Box H)$.

It is known that $\gamma(G) \cdot \gamma(H) \leq 2 \cdot \gamma(G\Box H)$ (Clark and Suen, 2000). The conjecture has been also verified for some graph classes.

In the talk we present some recent results related to this topic.

## A NOTE ON $k$-CORDIAL $p$-UNIFORM HYPERTREES

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For a $p$-uniform hypergraph $H = (V, E)$ and a $k$-labeling $c : V \to Z_k$ let $v_c(i) = |c^{-1}(i)|$. The coloring $c$ is said to be $k$-*friendly* if $|v_c(i) - v_c(j)| \leq 1$ for any $i \neq j; i, j \in Z_k$. The coloring $c$ induces an edge labeling $c^* : E \to Z_k$ defined by $c^*(e_j) = \sum_{x_i \in e_j} c(x_i) \mod k$. Let $e_{c^*}(i) = |{c^*}^{-1}(i)|$. A hypergraph is said to be $k$-*cordial* if it admits such $k$-friendly coloring $c$ that $|e_{c^*}(i) - e_{c^*}(j)| \leq 1$ for any $i \neq j; i, j \in Z_k$. Then we say that the edge coloring $c^*$ is $k$-*cordial.*

We show that any $p$-uniform hypertree $T$ is $k$-cordial for $k \in \{2, p-1, p\}$.

### REFERENCES

[1] I. Cahit, Cordial Graphs: a weaker version of graceful and harmonious graphs, Ars Combin. **32** (1987) 201–207

[2] N. Cairnie, K. Edwards, The computational complexity of cordial and equitable labelling, Discrete Math. **216** (2000) 29–34.

# INFORMATION-THEORETIC CHARACTERIZATION OF GRAPHS

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## 1. TOPIC AND RESULTS

To characterize graphs by using graph measures is an interesting problem in graph theory [3]. This can be done either by using simple graph measures and also by applying statistical and information-theoretic techniques [6]. In particular, a graph can be characterized using SHANNON's entropy [1, 2, 4, 5]. Starting from a recently developed approach to determine the structural information content of an unweighted graph based on using information functionals [2], we present a possible extension to weighted graphs. Moreover, we show some further developments of the method including graph entropy measure which are based on graph decompositions.

**Keywords:** Graphs, Graph Characterization, Entropy, Information Theory.
**AMS Subject Classification:** 05C99.

### REFERENCES

[1] D. Bonchev, Information Theoretic Indices for Characterization of Chemical Structures. Research Studies Press, Chichester, 1983.

[2] M. Dehmer, Information-theoretic Concepts for the Analysis of Complex Networks, Applied Artificial Intelligence Vol. 22 (7 & 8), 2008, 684–706

[3] D. Koschützki, K.A. Lehmann, L. Peters, S. Richter, D. Tenfelde-Podehl and O. Zlotkowski, Clustering, in: U. Brandes and T. Erlebach, editors, *Centrality Indices*, Lecture Notes of Computer Science, pages 16–61. Springer, 2005.

[4] A. Mowshowitz, Entropy and the complexity of the graphs I: An index of the relative complexity of a graph, Bull. Math. Biophys. 30, 1968, 175–204.

[5] N. Rashewsky, Life, Information Theory, and Topology, Bull. Math. Biophys., Vol. 17, 1955, 229–235.

[6] C.E. Shannon and W. Weaver, *The Mathematical Theory of Communication*, University of Illinois Press, 1997.

# ON THE PLANARITY AND OUTERPLANARITY OF ITERATED GRAPHS

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The middle index of a graph $G$ is the smallest $k$ such that $k$-th iterated middle graph of $G$ is non-planar. Similarly we define total, line-block, middle-block, outer-line, outer-middle, outer-total indices. In this paper we present characterizations of all graphs with respect to their middle, total, line-block, middle-block, outer-line, outer-middle and outer-total index.

**Keywords:** planarity, outerplanarity, line graph, middle graph, total graph.

**AMS Subject Classification:** 05C10.

## References

[1] M. Ghebleh, M. Khatirinejad, *Planarity of iterated graphs*, Discrete Math. 308 (2008) 144–147.

[2] V.R. Kulli, D.G. Akka, *Traversability and planarity of total-block graphs*, Jour. Math. Phy. Sci. Vol. 11, No. 4 (1977).

[3] J. Sedláček, *Some properties of interchange graphs*, in: M. Fiedler (ed.) Theory of Graphs and Its Applications, Publishing House of the Czechoslovak Academy of Sciences, Prague (1964) 145–150.

[4] M.M. Syslo, J. Topp, *On the planarity and outerplanarity of generalized linegraphs, generalized middle graphs, and generalized total graphs*, J. Combin. Inform. System Sci. 10 (1985) 69–78.

# ON THE FIBONACCI NUMBER OF CONNECTED CYCLE-SEPARATED GRAPHS

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Let $G$ be a graph. The total number of its independent sets is called the Fibonacci number or the Merrifield-Simmons index of the graph and it is denoted by $i(G)$. A connected cycle-separated graph is a connected graph which has no two cycles with a common vertex. If $G$ is a connected cycle-separated graph and the parameters $n$ and $m$ denote the number of its vertices and the number of its edges, respectively, then the number of cycles of $G$ is $m - n + 1$. We call a connected cycle-separated graph with $r$ cycles a connected cycle-separated graph of kind $r$. Obviously, $r \leq 2n - 5$. Therefore, trees are the connected cycle-separated graphs of kind 0. For connected cycle-separated graph of kind 0, $P_n$ (the path graph of length $n$) and $S_n$ (the star tree) have the smallest and the largest Fibonacci number, respectively [8, 7, 9, 6]. For connected cycle-separated graphs of kind 1 in [2, 5] the graphs with the smallest and the largest Fibonacci number have been characterized. The graphs with the smallest and largest Fibonacci number among the connected cycle-separated graphs of kind 2 have been characterized in [1, 3]. In this paper we investigate some increasing and decreasing transformations for Fibonacci number of connected cycle-separated graphs of arbitrary kind and then we characterize the extremal connected cycle-separated graphs with respect to the Fibonacci number.

**Keywords:** independent set, Fibonacci number of a graph, Merrifield-Simmons index, cycle-separated graph.

## REFERENCES

[1] H. Deng *The smallest MerrifieldŰSimmons index of (n,n+1)-graphs* Mathematical and Computer Modelling 49 (2009), pp. 320-326.

[2] H. Deng, S. Chen, *The extremal unicyclic graphs with respect to Hosoya index and Merrifield-Simmons index*, MATCH Commun. Math. Comput. Chem. **59** (2008) 171–190.

[3] H. Deng, S. Chen, J. Zhang, *The Merrifield-Simmons index in* $(n, n+1)$*-graphs*, J. Math. Chem **43** (2008) 75–91.

[4] R.E. Merrifield and H.E. Simmons, "Topological Methods in Chemistry" Wiley, New York, 1989.

[5] A.S Pedersen, P.D. Vestergaard, *The number of independent sets in unicyclic graphs*, Discrete Appl. Math. **152** (2005) 246–256.

[6] H. Prodinger, R.F. Tichy, *Fibinacci numbers of graphs*, Fibinacci Quart. **20** (1982) 16–21.

[7] I. Gutman and O.E. Polansky, *Mathematical Concepts in Organic Chemistry*, Springer, Berlin (1986).

[8] I. Gutman, *Acyclic systems with extremal Hückel $\pi$-electron*, Theor. Chim. Acta 45 (1977), pp. 79–87.

[9] X. Li, Z. Li and L. Wang, *The inverse problems for some topological indices in combinatorial chemistry*, J. Comput. Biol. 10 (2003), pp. 47–55.

# ALTITUDE OF $r$-PARTITE AND COMPLETE GRAPHS

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An edge-ordering of a graph $G = (V; E)$ is a one-to-one function $f : E(G) \rightarrow \{1, 2, \ldots, |E(G)|\}$. A path of length $k$ in $G$ is called a $(k, f)$-ascent if $f$ increases along the successive edges forming the path. The altitude $\alpha(G)$ of $G$ is the greatest integer $k$ such that for all edgeorderings $f$, $G$ has a $(k, f)$-ascent.

In our paper we give exact values or bounds of $\alpha(G)$ for some $r$-partite graphs. These results imply new bounds on $\alpha(K_n)$.

**AMS Subject Classification:** 05C78, 05C15, 05C38.

## REFERENCES

[1] Burger A.P., Cockayne E.J., Mynhardt C.M., Altitude of small complete and complete bipartite graphs, *Australas. J. Combin.* **31** (2005) 167–177.

[2] Dzido T., Furmańczyk H., Altitude of wheels and wheel-like graphs, submitted.

[3] Katrenič J., Semanišin G., Complexity of Ascent Finding Problem, *Proceedings of SOFSEM 2008*, High Tatras, Slovakia, January 20-24 2008, Vol. II 70–77.

## ON PLANAR TOEPLITZ GRAPHS

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An undirected graph $G = (V, E)$ is called *Toeplitz*, if its adjacency matrix $A$ is a Toeplitz matrix, i.e., a $0 - 1$ matrix whose entries are identical along the diagonals. A Toeplitz graph is therefore completely defined by the first row of A, a $0-1$ sequence. We present conditions on that sequence to define a *planar* Toeplitz graph, and study the chromatic number of these graphs. Finally, we adress the problem of counting maximal independent sets in planar Toeplitz graphs.

### REFERENCES

[1] R. Euler, Coloring planar Toeplitz graphs and the stable set polytope, Discrete Math. 276 (2004) 183-200.

[2] T. Zamfirescu, Some classes of finite, planar Toeplitz graphs, Working paper, 1994.

# VERTEX-ANTIMAGIC LABELINGS OF REGULAR GRAPHS

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joint work with Ali Ahmad, Kashif Ali, Martin Bača and Petr Kovář

Let $G = (V, E)$ be a finite, simple and undirected graph with $p$ vertices and $q$ edges. An $(a, d)$-vertex-antimagic total labeling is a bijection $f$ from $V(G) \cup E(G)$ into the set of consecutive integers $1, 2, \ldots, p + q$, such that the vertex-weights form an arithmetic progression with the initial term $a$ and the common difference $d$, where the vertex-weight of $x$ is the sum of values $f(xy)$ assigned to all edges $xy$ incident to vertex $x$ together with the value assigned to $x$ itself, i.e. $f(x)$. Such a labeling is called super if the smallest possible labels appear on the vertices.

In this talk, we study the properties of such labelings and examine their existence for $2r$-regular graphs for the values of difference $d = 0, 1, \ldots, r + 1$.

**Keywords:** super vertex-antimagic total labeling, vertex-antimagic edge labeling, regular graph.

**AMS Subject Classification:** 05C78.

# $\mathcal{Q}$-RAMSEY CLASSES OF GRAPHS

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Suppose $\mathcal{Q}$ is a hereditary graph property and assume $\mathcal{Q} \subseteq \mathcal{O}^2$, where $\mathcal{O}^2$ denotes the class of bipartite graphs. We define a $(\mathcal{Q}, k)$*-colouring* of a graph $G$ as a mapping $f : V(G) \rightarrow C$, where $C = \{1, \ldots, k\}$ is a set of *colours*, satisfying the condition that for every two distinct colours $i$ and $j$, the subgraph induced in $G$ by all the edges linking a vertex coloured with $i$ and a vertex coloured with $j$ belongs to $\mathcal{Q}$.

If we additionally assume that for every colour $i$ the set of vertices coloured with $i$ is independent, then these $(\mathcal{Q}, k)$-colourings are a natural generalization of acyclic colourings if $\mathcal{Q}$ is the class of acyclic graphs, star-forest colourings if $\mathcal{Q}$ is the class of star-forests, and so on.

Let $\mathcal{P}$ be a graph property and assume $k \geq 2$. We say that $\mathcal{P}$ is a $(\mathcal{Q}, k)$-*Ramsey class*, if for every $G \in \mathcal{P}$ there exists $H \in \mathcal{P}$ such that for every $(\mathcal{Q}, k)$-colouring of $H$ there is a colour $i$ such that $G \subseteq H[V_i]$, where $V_i$ is the set of vertices coloured with $i$.

The notion of $(\mathcal{Q}, 2)$-Ramsey classes of graphs was introduced in [2], as a generalization of vertex-Ramsey classes of graphs, see [4] for a survey.

In this talk we concentrate on $(\mathcal{Q}, k)$-Ramsey classes of graphs and we also mentioned some related problems.

**Keywords:** $\mathcal{Q}$-colouring, $\mathcal{Q}$-Ramsey class.

**2000 Mathematics Subject Classification:** 05C15, 05C55.

## References

[1] M. Borowiecki, I. Broere, M. Frick, P. Mihók and G. Semanišin, *A survey of hereditary properties of graphs*, Discuss. Math. Graph Theory **17** (1997) 5–50.

[2] M. Borowiecki, A. Fiedorowicz, *On partitions of hereditary properties of graphs*, Discuss. Math. Graph Theory **26** (2006) 377–387.

[3] B. Grűnbaum, *Acyclic coloring of planar graphs*, Israel J. Math. **14** (1973) 390–412.

[4] J. Nešetřil, *Ramsey Theory*, in: R.L. Graham, M. Grötschel, L. Lovász, eds., Handbook of Combinatorics, (North-Holland, 1995) 1331–1403.

# BOUNDS ON THE SIZE OF IDENTIFYING CODES FOR GRAPHS OF MAXIMUM DEGREE $\Delta$*

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Identifying codes in graphs are related to the classical notion of dominating sets and its locating variant [SR84]. They have a property which allows unique identification of all vertices of the graph. Identifying codes were first introduced in 1998 [KCL98], and have since been studied widely in the communities of both graph theory and coding theory.

Formally, given an undirected simple graph $G = (V, E)$, an *identifying code* is a subset $C \subseteq V$ such that $C$ is a dominating set of $G$, and for every pair of vertices $\{u, v\} \in V$ there exists $x \in C$ which dominates exactly one of the vertices of the pair $\{u, v\}$.

In this talk we discuss the relationship between the maximum degree $\Delta$ of a graph and the lower and upper bounds for the minimum cardinality of an identifying code in this graph. Such considerations are an extension of the known upper bound [GM07] of $n - 1$ on the size of the identifying code of an identifiable graph on $n$ vertices. Specifically, we show that any identifiable triangle-free graph $G$ has an identifying code of cardinality at most $n - \frac{n}{3\Delta+3}$, and of cardinality at most $n - \frac{n}{2\Delta+2}$ if it is $\Delta$-regular. We also present related bounds for graphs of girth at least 5.

## REFERENCES

[GM07] S. Gravier and J. Moncel. On graphs having a $V \setminus \{x\}$ set as an identifying code. *Discrete Mathematics*, 307(3-5) 432–434, 2007.

[KCL98] M.G. Karpovsky, K. Chakrabarty, and L.B. Levitin. On a new class of codes for identifying vertices in graphs. *IEEE Transactions on Information Theory*, 44 599–611, 1998.

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[SR84] P.J. Slater and D.F. Rall. On location-domination numbers for certain classes of graphs. *Congressus Numerantium*, 45 97–106, 1984.

## TRACEABILITY OF ORIENTED GRAPHS

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A digraph is $k$-*traceable* if each of its subdigraphs of order $k$ is traceable. All 2-traceable oriented graphs are traceable, since they are tournaments. We extend this result by showing that all $k$-traceable oriented graphs are traceable, for $k = 2, 3, 4, 5, 6$. However, for all $k \geq 7$, except possibly for $k = 8, 10$, there exist $k$-traceable oriented graphs that are nontraceable.

**Keywords:** Oriented graphs, traceable.

**AMS Subject Classification:** 05C20, 05C38.

## KERNELS BY MONOCHROMATIC PATHS AND THE CLASS COLOR DIGRAPH

H. Galeana-Sánchez

An $m$-coloured digraph is a digraph whose edges are coloured with $m$-colors. A directed path is monochromatic when its arcs are coloured alike.

A set $S \subseteq V(D)$ is a kernel by monochromatic paths whenever the two following conditions hold.

(1) For any $x, y \in S$, $x \neq y$ there is no monochromatic directed path between them.

(2) For each $z \in V(D) - S$ there exists a $zS$-monochromatic directed path. In this talk is introduced the concept of class color digraph to prove that if $D$ is an $m$-coloured digraph such that: (i) Every closed directed walk has an even number of changes of color. (ii) Every directed path starting and ending with the same color has an even number of changes of color. Then $D$ has a kernel by monochromatic paths.

This result generalizes widely a classical result by Sands, Sawer and Woodrow which asserts that any 2-coloured digraph has a kernel by monochromatic paths.

**Keywords:** Kernel; kernel by monochromatic path; the class color digraph.

**AMS Subject Classification:** 05C20, 05C15.

# ON MONOCHROMATIC PATHS AND QUASI-TRANSITIVE SUBDIGRAPHS IN ARC-COLOURED DIGRAPHS

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Let $D$ be a digraph, $V(D)$ and $A(D)$ will denote the set of vertices and arcs of $D$, respectively. We call the digraph $D$ an $m$-coloured digraph if the arcs of $D$ are coloured with $m$ colours. A directed path (or a directed cycle) is called *monochromatic* if all of its arcs are coloured alike. Now, let $D$ be an $m$-coloured digraph. A set $N \subseteq V(D)$ is said to be a ***kernel by monochromatic directed paths*** of $D$ if it satisfies the following two conditions: (i) for every pair of different vertices $u, v \in N$ there is no monochromatic directed path in $D$ between them and; (ii) for every vertex $x \in V(D) - N$ there is a vertex $y \in N$ such that there is an $xy$-monochromatic directed path in $D$.

A digraph is transitive whenever $(u,v) \in A(D)$ and $(v,w) \in A(D)$ implies $(u,w) \in A(D)$ or $(w,u) \in A(D)$. If $u \in V(D)$ we denote by $A^+(u)$ the set of arcs $\{(u,v) \in A(D) \mid v \in V(D)$ and we say that $A^+(u)$ is monochromatic if all of its elements have the same color.

Let $D$ be a $m-$coloured digraph such that there exist two spanning subdigraph of $D, D_1$ and $D_2$ such that: $F(D_1) \cap F(D_2) = \emptyset$, $F(D_1) \cup F(D_2) = F(D)$ and colors$(D_1)\cap$colors$(D_2) = \emptyset$. $C = (u_0, \ldots, u_k = v_0, \ldots, v_m = w_0, \ldots, w_n = u_0)$ will be called a $(D_1, D, D_2)$ subdivisions of $C_3$ $3-$coloured if $T_1 = (u_0, \ldots, u_k)$ is a monochromatic directed path of color $a$ and is contained in $D_1$, $T_2 = (v_0, \ldots, v_m)$ is a monochromatic directed path of color $b$ and is contained in $D$, and $T_3 = (w_0, \ldots, w_n)$ is a monochromatic directed path of color $c$ and is contained in $D_2$ with $a \neq b$, $b \neq c$, and $a \neq c$. And $P = (u_0, \ldots, u_k = v_0, \ldots, v_m = w_0, \ldots, w_n)$ will be called a $(D_1, D, D_2)$ subdivisions of $P_3$ $3-$coloured if $T_1 = (u_0, \ldots, u_k)$ is a monochromatic directed path of color $a$ and is contained in $D_1$, $T_2 = (v_0, \ldots, v_m)$ is a monochromatic directed path of color $b$ and is contained in $D$, and $T_3 = (w_0, \ldots, w_n)$ is a monochromatic directed path of color $c$ and is contained in $D_2$ with $a \neq b$, $b \neq c$, and $a \neq c$.

In this work it is proved that if $D$ is a $m-$coloured digraph such that there exists two spanning subdigraphs $D_1$ and $D_2$ of the digraph $D$ satisfied the following conditions: $F(D_1) \cap F(D_2) = \emptyset$, $F(D_1) \cup F(D_2) = F(D)$, $colors(D_1) \cap colors(D_2) = \emptyset$; $D_i$ be an $m$-coloured quasi-transitive digraph such that for every $u \in V(D_i)$, $A^+(u)$ is monochromatic, and $D_i$ has no $C_3$ 3-coloured for $i \in \{1,2\}$; $D$ does not contain $(D_1, D, D_2)$ subdivisions of $C_3$ $3-$coloured and if $(u,v,w,x)$ is a $(D_1, D, D_2)$ subdivision of $P_3$ $3-$coloured then there exists a monochromatic directed path in $D$ between $u$ and $x$. Then $D$ has a kernel by monochromatic directed paths.

**Keywords:** $m$-coloured digraph, quasi-transitive digrahs, kernel by monochromatic paths.

## References

[1] C. Berge, *Graphs*, North-Holland, Amsterdam, 1985.

[2] H. Galeana-Sánchez, *Kernels in edge-coloured digraphs*, Discrete Math. **184** (1998), 87–99.

[3] H. Galeana-Sánchez, *On monochromatic paths and monochromatics cycles in edge coloured tournaments*, Discrete Math. **156** (1996), 103–112.

[4] H. Galeana-Sánchez and R. Rojas-Monroy, *On monochromatic paths and monochromatic 4-cycles in edge-coloured bipartite tournaments*, Discrete Math. **285** (2004), 313–318.

[5] H. Galeana-Sánchez and R. Rojas-Monroy, *A counterexample to a conjecture on edge-coloured tournaments*, Discrete Math. **282** (1–3), (2004), 275–276.

[6] J. Bang-Jensen and J. Huang, *Quasi-transitive digraphs*, J. Graph Theory 20 (2) (1995), 141-161.

[7] H. Galeana-Sánchez, R. Rojas-Monroy and B. Zavala, *Monochromatic paths and monochromatic sets of arcs in quasi-transitive digraphs*, Submited.

[8] B. Sands, N. Sauer and R. Woodrow, *On monochromatic paths in edge-coloured digraphs*, J. Combin. Theory Ser. B **33** (1982), 271–275.

[9] Shen Minggang, *On monochromatic paths in $m$-coloured tournaments*, J. Combin. Theory Ser. B **45** (1988), 108-111.

# IN- AND EX-NEIGHBORS PRESERVING INDEPENDENCE: THE STRONG ARC-LOCALLY SEMICOMPLETE DIGRAPHS

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Jørgen Bang-Jensen introduced the arc-locally semicomplete digraphs as a common generalization of semicomplete and bipartite semicomplete digraphs in [1]. A digraph $D$ is *arc-locally semicomplete* if and only if whenever $x$ and $y$ are distinct vertices and there exists an arc $u \to v$ in $D$ such that $x \to u$ and $y \to v$ in $D$, there is at least one arc between $x$ and $y$ in $D$ and whenever $x$ and $y$ are distinct vertices and there exists an arc $u \to v$ in $D$ such that $u \to x$ and $v \to y$ in $D$, there is at least one arc between $x$ and $y$ in $D$. The previous definition has an interesting equivalence, when the digraphs are strong, wich is formulated in terms of independent sets of vertices:

**Proposition 1.** *A digraph $D$ is strong arc-locally semicomplete if and only if for all independent vertex set $S$ of $D$, both $N^-(S)$ and $N^+(S)$ are independent.*

In [2], Bang-Jensen claimed that the only strong arc-locally semicomplete digraphs are the extensions of cycles, the semicomplete digraphs and the bipartite semicomplete digraphs. But one family of strong arc-locally semicomplete digraphs is missing. Let $C_3^*$ be the digraph with vertex set $\{v_1, v_2, v_3\}$ and arc set $\{v_1v_2, v_2v_3, v_3v_1, v_1v_3\}$. It is easy to check that $C_3^*[E_1, E_n, E_1]$ is a family of strong arc-locally semicomplete digraphs, with the composition of digraphs as defined in [3] and where $E_i$ denotes the independent set of $i \geq 1$ vertices.

In this talk, we correct and extend the chacterization of Bang-Jensen to all arc-locally semicomplete digraphs.

**Keywords:** Arc-local tournament; Arc-locally semicomplete digraph, Generalization of tournaments; Independent set of vertices; Product of digraphs.

**AMS Subject Classification**: Primary: 05C20; Secondary: 05C75.

## References

[1] J. Bang-Jensen, *Arc-local tournament digraphs: a generalization of tournaments and bipartite tournaments*, Technical Report Preprint no. 2, Department of Mathematics and Computer Science, University of Southern Denmark, 1993.

[2] J. Bang-Jensen, *The structure of strong arc-locally semicomplete digraphs*, Discrete Math., **283** (2004), 1–6.

[3] J. Bang-Jensen and G. Gutin, "Digraphs: theory, algorithms, and applications," Monographs in mathematics, Springer, 2001.

# TURÁN NUMBERS FOR DISJOINT COPIES OF GRAPHS

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The *Turán number* $ex(n, \mathcal{H})$ of $\mathcal{H}$ is the maximum number of edges of an $n$-vertex simple graph having no member of $\mathcal{H}$ as a subgraph. We show lower and upper bounds for Turán numbers for disjoint copies of graphs. We also conjecture that the lower bound is sharp for disjoint paths $P_3$ and prove the conjecture in case of two and three $P_3$s.

**Keywords:** Turán number, extremal graph, disjoint copies.

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## ON PACKABLE DIGRAPHS

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One of the classical results in packing theory states that every graph of order $n$ and size less than or equal to $n-2$ is packable in its complement [1]. Moreover, the bound is sharp because the star is not packable. A similar problem arises for digraphs, namely, to find the maximal number $f_D(n)$ such that every digraph of order $n$ and size less than or equal to $f_D(n)$ is packable. So far it is known that $\frac{7}{4}n - 81 \leq f_D(n) \leq 2n - 4$ where the upper bound is sharp [3]. We show that $f_D(n) \geq 2n - o(n)$.

**Keywords:** packing, digraph.

**AMS Subject Classification:** 05C70, 05C20.

### References

[1] D. Burns and S. Schuster, Every $(p, p-2)$ graph is contained in its complement, J. Graph Theory 1 (1977) 277–279.

[2] A. Görlich, M. Pilśniak, M. Woźniak, I. A. Zioło, Fixed-point-free embeddings of digraphs with small size, Discrete Math. 307 (2007) 1332–1340.

# ABOUT OF THE STRUCTURE OF KERNEL PERFECT AND CRITICAL KERNEL-IMPERFECT DIGRAPHS

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A kernel of a directed graph is a set of vertices which is both independent and absorbent. A digraph is called critical kernel-imperfect if it has no kernel but every proper induced subdigraph has at least one. Berge and Duchet [Recent problems and results about kernels in directed graphs. Discrete Math. 86:27–31, 1990] proved that a critical kernel-imperfect digraph is strongly connected. In this work, CKI-digraphs are also proved to be 2-edge connected under certain requirements. Moreover, it is shown that by removing two vertices from a CKI-digraph on at least 5 vertices, the resulting digraph is not a transitive tournament. Sufficient conditions on the digraph are also given in order to guarantee that a digraph is kernel perfect.

**Keywords:** Kernel, kernel-perfect, critical kernel-imperfect, connectivity.

## REFERENCES

[1] Berge, C., *Nouvelles extensions du noyau d'un graphe et ses aplications en théorie des jeux*, Publ. Econométriques **6** (1973) 6–11.

[2] Berge, C., "Graphs," North Holland Mathematical Library 6: Chapter 14, 1985.

[3] Berge, C. and P. Duchet, *Recent problems and results about kernels in directed graphs.* Discrete Math. **86** (1990) 27–31.

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[4] Berge, C. and A. Ramachandra Rao, *A combinatorial problem in logic.* Discrete Math. **17** (1977) 23–26.

[5] Duchet, P. *Graphes Noyau-parfaits.* Ann. Discrete Math. **9** (1980) 93–101.

[6] Galeana-Sánchez, H., *Semikernels modulo F and kernels in digraphs*, Discrete Math. **218** (1-3) (2000), 61–71.

[7] Galeana-Sánchez, H and V. Neumann-Lara, *On Kernel-perfect critical digraphs.* Discrete Math. **59** (1986) 257–265.

[8] Geller, D. and F. Harary, *Connectivity in digraphs.* Lec. Not. Math. **186** (1970) Springer, Berlin, 105–114.

[9] Von Neumann, J. and O. Morgenstern, "Theory of Games and Economic Behavior." Princeton University Press, Princeton, 1944.

[10] Richardson, M. *Solutions of irreflexible relations.* Annals of Math. **58** (1953) 573–590.

# LOWER BOUNDS ON THE INDEPENDENCE NUMBER OF A GRAPH WITH GIVEN CLIQUE NUMBER

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New lower bounds on the independence number of a graph in terms of order, size, and clique number are presented. Their algorithmic realization is discussed.

## DOMINATION-TYPE PARAMETERS IN CUBIC GRAPHS

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In this talk we present various results on domination-type parameters in cubic graphs.

# $k$-KERNELS IN GENERALIZATIONS OF TOURNAMENTS

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Let $D$ be a digraph with arc set $A(D)$ and vertex set $V(D)$. A set $S \subseteq V(D)$ is said to be $k$-independent if for every $u, v \in S$, $d_D(u, v) \geq k$; $S$ is said to be $l$-absorbent if for every $u \in V(D) \setminus S$ exists $v \in S$ such that $d_D(u, v) \leq l$. A set $N \subseteq V(D)$ is called a $(k, l)$-kernel if it is $k$-independent and $l$-absorbent. A $(k, k-1)$-kernel is a $k$-kernel; $k$-semikernels are defined analogously. Since a kernel is a 2-kernel, $k$-kernels are generalizations of kernels. Inspired in a classic result due to Neumann-Lara [11], we introduce new sufficient conditions for certain families of digraphs to have a $k$-kernel using the fact that they have non-empty $k$-semikernel. Generalizations of tournaments studied are: local in, local out, arc local in, arc local out, left pretransitive, right pretransitive, $k$-transitive and quasi-transitive digraphs. It's proved that every quasi-transitive digraph has a $k$-kernel for $k \geq 3$. All results remain valid for infinite digraphs, but in quasi-transitive digraphs we must ask for an additional condition, the digraph must not contain infinite outward paths.

**Keywords:** kernel, $k$-kernel, $(k, l)$-kernel, quasi-transitive, generalized tournaments.

**AMS Subject Classification:** 05C20, 05C38, 05C69.

## References

[1] Bang-Jensen J., J. Huang, and E. Prisner, In-tournament digraphs, J. Combin. Theory (B) **59(2)** (1993) 267–287.

[2] Bang-Jensen J., G. Gutin, *Generalizations of tournaments: A survey*, Journal of Graph Theory, **28**:171202, 1998.

[3] Bang-Jensen J., Y. Guo, G. Gutin, and L. Volkmann, *A classification of locally semicomplete digraphs*, Discrete Math. **167/168** (1997) 101–114. 15th British Combinatorial Conference (Stirling, 1995).

[4] Bang-Jensen J., *Arc-local tournament digraphs: a generalization of tournaments and bipartite tournaments*, Technical report 2, Department of Mathematics and Computer Science, Odense University, Denmark, 1993.

[5] Bang-Jensen J., J. Huang, *Quasi-transitive digraphs.* J. Graph Theory, **20(2)** (1995) 141–161. December 1982, Pages 271–275.

[6] Bang-Jensen J., G. Gutin, *Digraphs. Theory, Algorithms and Applications*, Springer-Verlag, 2002.

[7] Berge, C. *Some classes of perfect graphs*, Graph Theory and Theoretical Physics (Academic Press, London, 1967) 155–165, MR 38 No. **1017**.

[8] Boros, E., V. Gurvich, *Perfect Graphs are Kernel Solvable*, Discrete Mathematics **159(1-3)** (1996) 35–55.

[9] Chudnovsky M., N. Robertson, P. Seymour, R. Thomas, *The Strong Perfect Graph Theorem*, Annals of Mathematics **164** (2006) 51–229.

[10] Galeana-Sánchez H., V. Neumann-Lara., *On kernels and semikernels of digraphs*, Discrete Mathematics **48** (1984) 67–76. North-Holland.

[11] Neumann-Lara V., *Seminúcleos de una digráfica*, Anales del Instituto de Matemáticas **II**, UNAM, 1971.

[12] Waldemar S., A. Włoch, I. Włoch, *On $(k,l)$-kernels in $D$-join of digraphs*, Discuss. Math. Graph Theory **27** (2007) 457–470.

# PARITY VERTEX COLOURING OF GRAPHS

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A parity path in a vertex colouring of a graph is a path along which each colour is used an even number of times. Let $\chi_p(G)$ be the least number of colours in a vertex colouring of $G$ having no parity path. It is proved that for any graph $G$ there is

$$\chi(G) \leq \chi_p(G) \leq |V(G)| - \alpha(G) + 1$$

where $\chi(G)$ and $\alpha(G)$ is the chromatic number and the independence number of $G$, respectively. The bounds are tight. This result is improved for trees. Namely, if $T$ is a tree with diameter $diam(T)$ and radius $rad(T)$, then

$$\lceil \log_2 (2 + diam(T)) \rceil \leq \chi_p(T) \leq 1 + rad(T)\,.$$

The bounds are tight.

We will discuss a relation between the parity vertex cpolouring and the vertex ranking of graphs

## REFERENCES

[1] K. Budajová, S. Jendroľ and S. Krajči, *Parity vertex colouring of graphs.*

# GLOBAL SECURE SETS IN COGRAPHS

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Let $G = (V, E)$ be a graph. A global secure set $SD \subseteq V$ is a dominating set which satisfies an additional condition which says that $|N[X] \cap SD| \geq |N[X] - SD|$ for every subset $X \subseteq SD$. We present some results concerning global secure sets in cographs. Furthermore we will investigate whether, for a given graph $G$, from the existence of the global secure set of cardinality $k$ ($k \leq |V(G)| - 1$) follows the existence of the global secure set of cardinality $k + 1$.

**Keywords:** graph, alliance, secure set, dominating set, cograph, cotree.

**AMS Subject Classification:** 05C69, 05C05.

## References

[1] R.C. Brigham, R.D. Dutton, S.T. Hedetniemi, Security in graphs, Discrete Applied Math. 155 (2007) 1708–1714.

[2] M. Chellali, T.W. Haynes, Global Alliances and Independence in Trees, Discuss. Math. Graph Theory 27 (2007) 19–27.

[3] D.G. Corneil, Y. Perl, L.K. Stewart. A linear time recognition algorithm for cographs. J. Computing 14 (1985) 924–934.

[4] R.D. Dutton, R. Lee, R.C. Brigham, Bounds on graph's security number, Discrete Applied Math. 156 (2008) 695–704.

[5] T.W. Haynes, S.T. Hedetniemi, M.A. Henning, Global defensive alliances in graphs, Electron. J. Combin. 10 (1) (2003) R47.

[6] P. Heggernes, D. Kratsch, Linear-time certifying algorithms for recognizing trivially perfect graphs, Reports In Informatics ISSN 0333-3590, 339 (2006).

[7] P. Kristiansen, S.M. Hedetniemi, S.T. Hedetniemi, Alliances in graphs, J. Combin. Math. Comput. 48 (2004) 157–177.

# $d$-STRONG EDGE COLORINGS OF GRAPHS

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If $c : E \to \{1, 2, \ldots, k\}$ is a proper edge coloring of a graph $G = (V, E)$ then the palette $S(v)$ of a vertex $v \in V$ is the set of colors of the incident edges: $S(v) = \{c(e) : e = vw \in E\}$. An edge coloring $c$ distinguishes vertices $u$ and $v$ if $S(u) \neq S(v)$. A $d$-strong edge coloring of $G$ is a proper edge coloring that distinguishes all pairs of vertices $u$ and $v$ with distance $d(u, v) \leq d$. The minimum number of colors of a $d$-strong edge coloring is called $d$-strong chromatic index $\chi'_d(G)$ of $G$.

We prove some general bounds for $\chi'_d(G)$, determine $\chi'_d(G)$ completely for paths and give exact values for cycles disproving a general conjecture of Zhang et al. [3].

**Keywords:** edge colorings, strong chromatic index.

**AMS Subject Classification:** 05C15.

## References

[1] S. Akbari, H. Bidkhori, N. Nosrati: *$r$-Strong edge colorings of graphs.* Discrete Math. **306** (2006), 3005–3010.

[2] J.J. Tian, X. Liu, Z. Zhang, F. Deng: *Upper bounds on the $D(\beta)$-vertex-distinguishing edge-chromatic-numbers of graphs.* LNCS **4489** (2007), 453–456.

[3] Z. Zhang, J. Li, X. Chen et al.: *$D(\beta)$ vertex distinguishing proper edge coloring of graphs.* Acta Mathematica Sinica Chinese Series **49** (2006), 703–708.

# THERE ARE AT LEAST 660 UNIVERSAL ONE-SIXTHS OF $K_{17}$

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We consider an edge-decomposition of a complete n-vertex graph $K_n$ into $t$, $t = 6$, isomorphic parts so that a possible edge-remainder $R$ is as small as possible. Namely, $|R| = \binom{n}{2} \bmod 6$. The general conjecture [3] says that for any $t$ and any $n$ there exists a graph $F$ such that, for each edge-remainder $R$ of the smallest possible size, the graph $K_n - R$ is edge-decomposable into $t$ copies of $F$. Such a graph $F$ is called a universal $t$th part of $K_n$. Recently we have proved the conjecture in case $t = 6$. We consider the case of $K_{17}$ (with $t = 6$, $n = 17$ and $|R| = 4$) to show that the number of universal parts can be multiplied.

**Keywords:** minimum remainder, edge-decomposition, sixth part, decomposition matrix.

**AMS Subject Classification:** 05C70, 05C50.

## References

[1] A. Kędzior, Z. Skupień, *Universal sixth parts of a complete graph exist*, to appear.

[2] Z. Skupień, *The complete graph t-packings and t-coverings*, Graphs Comb. **9** (1993), 353-363.

[3] Z. Skupień, *Clique parts independent of remainders*, Discuss. Math. Graph Theory **22** (2002), 361.

[4] Z. Skupień, *Universal fractional parts of a complete graph*, to appear.

# ON CARTESIAN PRODUCTS OF CYCLES AND THEIR CROSSING NUMBERS

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The *crossing number* $cr(G)$ of a simple graph $G = (V, E)$ is defined as the minimum number of crossings among all possible projections of $G$ on the plane. Computing the crossing number of a given graph is in general an elusive problem. Garey and Johnson [2] have proved that this problem is NP–complete. According to their structure, Cartesian products of special graphs are one of few graph classes for which the exact values of crossing numbers were obtained. Let $C_n$ be the cycle on $n$ vertices. In 1973, Harary, Kainen, and Schwenk [4] established the crossing number of $C_3 \times C_3$ and conjectured that $cr(C_m \times C_n) = m(n-2)$ for $3 \leq m \leq n$. Recently has been proved by Glebsky and Salazar [3] that for any fixed $m \geq 3$, the conjecture holds for all $n \geq m(m+1)$.

The crossing numbers of the Cartesian products of cycles and all graphs of order four are determined in [1, 5]. The table in [6] shows the summary of known crossing numbers for Cartesian products of cycles and connected graphs of order five. We extend these results and we give the crossing numbers for the Cartesian products of the cycle $C_n$ with the specific graphs on six vertices.

**Keywords:** graph, drawing, Cartesian product, crossing number, cycle.

**AMS Subject Classification:** 05C10.

## References

[1] L.W. Beineke, R.D. Ringeisen, On the crossing numbers of products of cycles and graphs of order four, *J. Graph Theory* **4** (1980), 145–155.

[2] M.R. Garey, D.S. Johnson, Crossing number is NP-complete, *SIAM J. Algebraic and Discrete Methods* 4 (1983), 312–316.

[3] L.Y. Glebsky, G. Salazar, The crossing number of $C_m \times C_n$ is as conjectured for $n \geq m(m+1)$, *J. Graph Theory* 47 (2004), 53.72.

[4] Harary, F., P. C. Kainen, and A. J. Schwenk, *Toroidal graphs with arbitrarily high crossing numbers*, Nanta Math. **6** (1973), 58–67.

[5] S. Jendrol', M. Ščerbová, On the crossing numbers of $S_m \times P_n$ and $S_m \times C_n$, *Časopis pro pěstování matematiky* **107** (1982), 225–230.

[6] M. Klešč, The crossing numbers of Cartesian products of paths with 5-vertex graphs, *Discrete Mathematics* **233** (2001), 353–359.

## DOMINATION IN A DIGRAPH AND IN ITS REVERSE

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Let $D$ be a digraph. By $\gamma(D)$ we denote the domination number of $D$ and by $D^-$ we denote a digraph obtained by reversing all the arcs of $D$. In [1] the authors prove
Theorem A: If $D$ is a digraph of order $n \geq 2$ with no isolated vertices, then

$$2 \leq \gamma(D) + \gamma(D^-) \leq \frac{4n}{3},$$

and both the bounds are sharp.

While Theorem A bounds the sum of $\gamma(D)$ and $\gamma(D^-)$, we study their difference. The greatest difference $\gamma(D^-) - \gamma(D)$ is $n-2$ as is shown by orientation of $K_{1,n-1}$ if we direct all the arcs from the center. The problem is that this digraph is not strongly connected and its total domination number is $\infty$. We show that the difference $\gamma(D^-) - \gamma(D)$ cannot be bounded by a constant, even if we restrict to strongly connected regular digraphs. We prove that for every $\delta \geq 3$ and $k \geq 1$ there exists a simple strongly connected $\delta$-regular digraph $D_{\delta,k}$ such that $\gamma(D^-_{\delta,k}) - \gamma(D_{\delta,k}) = k$. Analogous theorem is obtained for total domination number provided that $\delta \geq 4$.

**Keywords:** domination number, total domination number, directed graph, reverse digraph, converse.

**AMS Subject Classification:** 05C69, 05C20.

### REFERENCES

[1] G. Chartrand, F. Harary, B.Q. Yue, On the out-domination and in-domination numbers of a digraph, Discrete Math. 197 (1999) 179-183.

# SOME RESULTS FOR THE WEAKLY CONVEX AND CONVEX DOMINATION NUMBERS OF A GRAPH

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For a connected graph $G = (V, E)$ with $|V| = n$, the ***neighbourhood*** of a vertex $v \in V$ in $G$ is the set $N_G(v)$ of all vertices adjacent to $v$ in $G$. For a set $X \subseteq V$, ***the open neighbourhood*** $N_G(X)$ is defined to be $\bigcup_{v\in X} N_G(v)$ and ***the closed neighbourhood*** $N_G[X] = N_G(X) \cup X$. A set $D \subseteq V$ is a ***dominating set*** of $G$ if $N_G[D] = V$. The ***distance*** $d_G(u, v)$ between two vertices $u$ and $v$ in a connected graph $G$ is the length of the shortest $(u-v)$ path in $G$. A $(u-v)$ path of length $d_G(u, v)$ is called $(u - v)$-***geodesic***. A set $X$ is ***convex*** in $G$ if vertices from all $(a - b)$-geodesic belong to $X$ for every two vertices $a, b \in X$. A set $X \subseteq V$ is a ***convex dominating set*** if $X$ is convex and dominating. The ***convex domination number*** $\gamma_{con}(G)$ of a graph $G$ is the minimum cardinality of a convex dominating set in $G$. Here we consider the Nordhaus-Gaddum results for the weakly convex domination number of a graph. Weakly convex and convex domination numbers of a cartesian product of some classes of graphs is also considered. We also investigate the influence of deleting an edge and an edge subdivision on the convex domination number.

**Keywords:** weakly convex domination number, convex domination number, Nordhaus-Gaddum results, edge subdivision, deleting an edge, cartesian product.

**AMS Subject Classification:** 05C05, 05C69.

## References

1. T. Haynes, S. Hedetniemi, P. Slater, *Fundamentals of domination in graphs*, Marcel Dekker, Inc. (1998).
2. J. Topp, P.D. Vestergaard, *Interpolation theorems for domination numbers of a graph*, Discrete Mathematics 191 (1998), 207-221.
3. J. Topp, Personal communication, Gdańsk University of Technology, 2002.
4. J. Cyman, M. Lemańska, J. Raczek, *Graphs with convex domination number close to their order*, Discuss. Math. Graph Theory **26** (2006) 307–316.

# THE DICHROMATIC NUMBER AND THE ACYCLIC DISCONNECTION IN TOURNAMENTS

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We disprove the following conjecture due to Víctor Neumann-Lara: for every couple of integers $(r, s)$ such that $r \geq s \geq 2$ there is an infinite set of circulant tournaments $T$ such that the dichromatic number and the acyclic disconnection of $T$ are equal to $r$ and $s$ respectively. We show that for every integer $s \geq 4$ there exists a lower bound $b(s)$ for the dichromatic number $r$ such that $\mathcal{F}_{r,s} = \varnothing$ ($\widetilde{\mathcal{F}}_{r,s} = \varnothing$ resp.) for every $r < b(s)$. We construct an infinite set of circulant tournaments $T$ such that $dc(T) = b(s)$ and $\overrightarrow{\omega}_3(T) = s$ ($\overrightarrow{\omega}(T) = s$ resp.) and give an upper bound $B(s)$ for the dichromatic number $r$ such that for every $r \geq B(s)$ there exists an infinite set $\mathcal{F}_{r,s}$ ($\widetilde{\mathcal{F}}_{r,s}$ resp.) of circulant tournaments. Some infinite sets $\mathcal{F}_{r,s}$ ($\widetilde{\mathcal{F}}_{r,s}$ resp.) of circulant tournaments are given for $b(s) < r < B(s)$.

**Keywords:** Circulant tournament, dichromatic number, acyclic disconnection.

**AMS Subject Classification:** 05C20.

## References

[1] B. Llano and V. Neumann-Lara, *Circulant tournaments of prime order are tight,* Discrete Math. **308** (2008) 6056-6063.

[2] B. Llano and M. Olsen, *Infinite families of tight regular tournaments,* Discuss. Math. Graph Theory **27** (2007) 299-311.

[3] B. Llano and M. Olsen, *On a conjecture of Víctor Neumann-Lara* (submitted).

[4] V. Neumann-Lara, *The acyclic disconnection of a digraph*, Discrete Math. **197/198** (1999) 617-632.

# PARTITIONING A GRAPH INTO A DOMINATING SET, A TOTAL DOMINATING SET, AND SOMETHING ELSE

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A recent result of Henning and Southey (A note on graphs with disjoint dominating and total dominating set, *Ars Comb.* **89** (2008), 159-162) implies that every connected graph $G$ of minimum degree at least 3 has a dominating set $D$ and a total dominating set $T$ which are disjoint. We show that the Petersen graph is the only such graph for which $D \cup T$ necessarily contains all vertices of the graph $G$.

**Keywords:** Domination; total domination; domatic number; vertex partition; Petersen graph.

**AMS Subject Classification:** 05C69.

## References

[1] N.J. Calkin and P. Dankelmann, The Domatic Number of Regular Graphs, *Ars Combinatoria* **73** (2004), 247-255.

[2] G.S. Domke, J.E. Dunbar, and L.R. Markus, The inverse domination number of a graph, *Ars Combin.* **72** (2004), 149-160.

[3] U. Feige, M.M. Halldórsson, G. Kortsarz, and A. Srinivasan, Approximating the Domatic Number, *SIAM J. Comput.* **32** (2002), 172-195.

[4] C. Godsil and G. Royle, Algebraic Graph Theory, Springer, 2001.

[5] T.W. Haynes, S.T. Hedetniemi, and P.J. Slater, Fundamentals of Domination in Graphs, Marcel Dekker, New York, 1998.

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[6] T.W. Haynes, S.T. Hedetniemi, and P.J. Slater (eds), Domination in Graphs: Advanced Topics, Marcel Dekker, New York, 1998.

[7] S.M. Hedetniemi, S.T. Hedetniemi, R.C. Laskar, L. Markus, and P.J. Slater, Disjoint dominating sets in graphs, manuscript 2007.

[8] M.A. Henning, C. Löwenstein, and D. Rautenbach, Remarks about Disjoint Dominating Sets, manuscript 2008.

[9] M.A. Henning and J. Southey, A note on graphs with disjoint dominating and total dominating set, *Ars Comb.* **89** (2008), 159-162.

[10] V.R. Kulli and S.C. Sigarkanti, Inverse domination in graphs, *Nat. Acad. Sci. Lett.* **14** (1991), 473-475.

[11] C. Löwenstein and D. Rautenbach, Pairs of Disjoint Dominating Sets and the Minimum Degree of Graphs, manuscript 2008.

[12] O. Ore, *Theory of graphs*, *Amer. Math. Soc. Transl.* **38** (Amer. Math. Soc., Providence, RI, 1962), 206-212.

[13] B. Zelinka, Total domatic number and degrees of vertices of a graph, *Math. Slovaca* **39** (1989), 7-11.

[14] B. Zelinka, Domatic numbers of graphs and their variants: A survey, in: *Domination in Graphs: Advanced Topics*, T.W. Haynes et al. eds, Marcel Dekker, New York, 1998, 351-377.

## ON DOUBLY LIGHT GRAPHS

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A graph $H$ is ***doubly light*** (***weakly doubly light***) in a family $\mathcal{H}$ of plane graphs if there exist finite numbers $a, b$ such that each graph $G \in \mathcal{H}$ which contains as a subgraph a copy of $H$, contains also a subgraph $K \cong H$ such that $\deg_G(x) \le a$ for each vertex $x$ of $K$, and $\deg_G(\alpha) \le b$ for each face of $G$ incident with a vertex of $K$ (incident with an edge of $K$, respectively). We present an overview of results on doubly and weakly doubly light graphs; in particular, we show that selected small graphs ($K_1, K_2$, short paths and cycles) are doubly or weakly doubly light if the corresponding family $\mathcal{H}$ is determined by conditions of prescribed minimum vertex/face degree or minimum edge/dual edge weight.

**Keywords:** plane graph, light graph, configuration.

**AMS Subject Classification:** 05C10.

## INTERVAL INCIDENCE GRAPH COLORING

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For a given simple graph $G = (V, E)$, we define an *incidence* as a pair $(v, e)$, where vertex $v \in V(G)$ is one of the ends of edge $e \in E(G)$ (we say $v$ is incident with $e$). Let us define a set of incidences $I(G) = \{(v, e) : v \in V(G) \wedge e \in E(G) \wedge v \in e\}$. We say that two incidences $(v, e)$ and $(w, f)$ are *adjacent* if one of the following holds: $(i)$ $v = w$, $e \neq f$, $(ii)$ $e = f$, $v \neq w$, $(iii)$ $e = \{v, w\}$, $f = \{w, u\}$ and $v \neq u$. By an *incidence coloring* of $G$ we mean a function $c : I(G) \to \mathbb{N}$ such that $c((v, e)) \neq c((w, f))$ for any adjacent incidences $(v, e)$ and $(w, f)$.

A finite subset $A$ of $\mathbb{N}$ is an *interval* if and only if it contains all numbers between $\min A$ and $\max A$, $|A| = \max A - \min A + 1$. For a given incidence coloring $c$ of graph $G$ let $A_c(v) = \{c((v, e)) : v \in e \wedge e \in E(G)\}$. By an *interval incidence coloring* of graph $G$ we mean an incidence coloring $c$ of $G$ such that for each vertex $v \in V(G)$ set $A_c(v)$ is an interval.

In this talk we consider a new model of incidence coloring of graphs and survey its general properties including lower and upper bounds on the number of colors. We present some polynomial-time algorithms for selected classes of graphs (e.g. bounded degree, bipartite). We are interested in determining computationally easy and hard instances.

**Keywords:** interval coloring, incidence coloring, star arboricity.

### REFERENCES

[1] Brualdi R.A., Massey J.Q., Incidence and strong edge colorings of graphs, *Discrete Mathematics*, 122 (1993), 51 – 58.

[2] Dolama M.H., Sopena E., Zhu X., Incidence coloring of k-degenerated graphs, *Discrete Mathematics*, 283 (2004), 121 – 128.

[3] Guiduli B., On incidence coloring and star arboricity of graphs, *Discrete Mathematics*, 163 (1997), 275 – 278.

[4] Janczewski R., Małafiejska A., Małafiejski M., Interval incidence coloring of graphs, *Zesz. Nauk. Pol. Gd.*, 13 (2007), 481 – 488.

## LEVEL HYPERGRAPHS

Hortensia Galeana-Sánchez and Martín Manrique

Given a hypergraph $H = (E_1, ..., E_m)$, its level-hypergraph $L_H$ is the result of identifying all vertices which belong to exactly the same edges. This new hypergraph has the same edge-structure as the original one, but may have less vertices. The tool makes it possible to emulate known theorems given in terms of order or rank; the new results are stated in terms of edge-structure, and usually apply to different classes of hypergraphs than the original statements, though there are some improvements on known results.

On the other hand, the study of several characteristics of a given hypergraph $H$ is simplified, since many hypergraph invariants are preserved. For example: $H$ is simple if, and only if, $L_H$ is simple; $H$ has repeated edges if, and only if, $L_H$ does too; $\nu(H) = \nu(L_H)$, where $\nu(H)$ is the maximum cardinality of a matching in $H$; the minimum cardinality of a transversal set, the maximum cardinality of a transversal set not contained properly in other transversal, and the minimum cardinality of a strongly stable set are also equal in both $H$ and $L_H$. Moreover, $H$ is balanced (respectively totally balanced) if, and only if, $L_H$ is balanced (respectively totally balanced); $H$ is unimodular (respectively strongly unimodular) if, and only if, $L_H$ is unimodular (respectively strongly unimodular), and $\Delta(H) = \Delta(L_H)$, $\delta(H) = \delta(L_H)$.

**Keywords:** hypergraph, balanced hypergraph, transversal set.

**AMS Subject Classification:** 05C20, 05C65, 05C69.

### References

-B. D. Acharya, *Unpublished Manuscript*, MRI, 1979.

-C. Berge, *The Theory of Graphs,* Dover Publications, New York 2001.

-C. Berge, *Hypergraphs. Combinatorics of Finite Sets,* Elsevier Science Publishers, Amsterdam 1989.

# GENERALIZED CIRCULAR COLOURING OF GRAPHS

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joint work with Janka ORAVCOVÁ and Roman SOTÁK

Let $\mathcal{P}$ be an additive and hereditary property of graphs and $r, s \in \mathbb{N}$, $r > s$. A circular $(\mathcal{P}, r, s)$-colouring of a graph $G$ is an assignment $f : V(G) \to [0, r-1]$, such that edges of $G$, consisting of vertices $u, v \in V(G)$, for which $|f(u) - f(v)| < s$ or $|f(u) - f(v)| > r - s$, induce a subgraph of a graph $G$ with the propery $\mathcal{P}$. In this talk we present some basic results on circular $(\mathcal{P}, r, s)$-colourings. We introduce the circular $\mathcal{P}$-chromatic number of a graph and we determine the circular $(\mathcal{P})$-chromatic number of complete graphs for additive and hereditary properties.

# LINEAR TIME ALGORITHM FOR FINDING LONG CYCLES IN STRONGLY CONNECTED 3-UNIFORM HYPERGRAPHS

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By a cycle of length $k$ in a hypergraph $H$ we mean an alternating sequence $(v_0, e_1, v_1, e_2, v_2, ..., v_{k-1}, e_k, v_k)$ of vertices and edges of this hypergraph satisfying the following conditions: (i) $e_i = e_j \Rightarrow i = j$ for $i, j = 1, ..., k$; (ii) $\{v_{i-1}, v_i\} \subseteq e_i$ for $i = 1, ..., k$; (iii) $v_{i-1} \neq v_i$ for $i = 1, ..., k$; (iv) $v_0 = v_k$. We consider cycles containing all edges of a hypergraph (long cycles).

Problems of this kind arise in computer graphics and geographic information systems.

We proved earlier that a strongly connected hypergraph $H = (V, E)$ has a long cycle iff $\sum_{v \in V} \lfloor d(v)/2 \rfloor \geqslant |E|$. In this talk we give a linear time algorithm for finding such a cycle in a 3-uniform hypergraph satisfying this condition.

# CONDUCTORS AND THE FROBENIUS VECTOR IN THE SYLVESTER-FROBENIUS CHANGE PROBLEM

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Let $\mathcal{A}$ be a set of $n$-dimensional integral vectors generating the lattice $\mathbb{Z}^n$. Let $C = C(\mathcal{A})$ be the cone generated by $\mathcal{A}$. A vector $\mathbf{v}$ is called $\mathcal{A}$-*reachable*, if $\mathbf{v}$ is a nonnegative integral linear combination of elements of $\mathcal{A}$. A minimal, with respect to cone ordering, integral vector $\mathbf{h}$ ($\mathbf{g}$) such that every integral vector in $C + \mathbf{h}$ (the interior of $C + \mathbf{g}$) is reachable is called a *conductor* (a *modular Frobenius vector*). Detailed treatment of the case $|\mathcal{A}| \leq n + 1$ will be presented, including relations between reachable and non-reachable vectors and formulas for the vectors $\mathbf{g}$ and (where possible) $\mathbf{h}$. Some extencions of the problem will be also discussed.

## References

1. M.J. Knight, *A generalization of a result of Sylvester's*, J. Number Theory 12 (1980), 364-366,
2. P. Pleasants, H. Ray, J. Simpson, *The Frobenius problem on lattices*, Australas. J. Combin. 32 (2005), 27-45,
3. J.L. Ramírez Alfonsín, *The Diophantine Frobenius Problem*, Oxford University Press, 2005,
4. B. Vizvári, *Beiträge zum Frobenius-Problem*, D.Sc. Nat. Dissertation, Technische Hohschule Carl Schorlemmer, Leuna-Merseburg, Germany (1987).

## A NOTE ON STAR PRODUCT OF GRAPHS AND GENERALIZED VOLTAGE ASSIGNMENTS

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Graph coverings, viewed through the optics of ordinary and permutation voltage assignments and associated lifts, have been successfully used in a number of recent constructions of large graphs and digraphs of given degree and diameter. In our contribution we review a few basic facts on ordinary and permutation voltage assignments and lifts with regard to the degree-diameter problem. We also suggest a new type of assignments that generalize ordinary voltages but are still a subclass of permutation voltages. We relate this new type of assignments to known constructions, in particular to the star product of graphs.

# CARTESIAN DIMENSION OF A GRAPH

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As suggested recently by G. Sabiddussi (personal communication) the natural Cartesian dimension of a graph $G$ is the smallest natural number $k$ with the property that there exist an induced embedding of $G$ into a Cartesian product of $k$-paths $P_{n_1}\Box P_{n_2}\Box\ldots\Box P_{n_k}$ in contrast to lattice dimension (isometric embedding) and ordinary embedding (with no additional conditions). We will present some edge coloring conditions for a graph $G$ that force such an embedding. This method is a specialization of some previous result of induced embedding in general Cartesian product graphs, see [1]. Also some exact results will be presented.

**Keywords:** induced subgraph; edge coloring; Cartesian product.

## REFERENCES

[1] I. Peterin, Characterizing flag graphs and induced subgraphs of Cartesian product graphs, Order 21 (2004) 283–292.

# INTERVAL EDGE COLORINGS OF SOME PRODUCTS OF GRAPHS

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An edge coloring of a graph $G$ with colors $1, 2, \dots, t$ is called an interval $t-$coloring [1] if at least one edge of $G$ is colored by $i, i = 1, 2, \dots, t$, the colors of edges incident to each vertex of $G$ are distinct and form an interval of integers. Let $\mathfrak{N}$ be the set of all interval colorable graphs. In [2] Giaro showed that if $G, H \in \mathfrak{N}$ then the Cartesian product of these graphs belongs to $\mathfrak{N}$. In the same paper he formulated a similar problem for the lexicographic product as an open problem. In this work we first show that if $G \in \mathfrak{N}$ then $G[nK_1] \in \mathfrak{N}$ for any $n \in N$. Further, we show that if $G, H \in \mathfrak{N}$ and $H$ is a regular graph then $G[H] \in \mathfrak{N}$. We also prove that tensor and strong tensor products of graphs $G, H$ belong to $\mathfrak{N}$ if $G \in \mathfrak{N}$ and $H$ is a regular graph.

**Keywords:** edge coloring, interval coloring, products of graphs.

**AMS Subject Classification:** 05C15.

## REFERENCES

[1] A.S. Asratian, R.R. Kamalian, Interval colorings of edges of a multigraph, Appl. Math. 5 (1987) 25-34 (in Russian).

[2] M. Kubale, Graph colorings, Amer. Math. Soc. (2004) 108-109.

# COLORING CHIP CONFIGURATIONS ON GRAPHS AND DIGRAPHS


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Let $D$ be a simple directed graph. Suppose that each edge of $D$ is assigned with some number of chips. For a vertex $v$ of $D$, let $q^+(v)$ and $q^-(v)$ be the total number of chips lying on the arcs outgoing form $v$ and incoming to $v$, respectively. Let $q(v) = q^+(v) - q^-(v)$. We prove that there is always a chip arrangement, with one or two chips per edge, such that $q(v)$ is a proper coloring of $D$. We also show that every undirected graph $G$ can be oriented so that adjacent vertices have different balanced degrees (or even different in-degrees). The arguments are based on peculiar chip shifting operation which provides efficient algorithms for obtaining the desired chip configurations. We also investigate modular versions of these problems. We prove that every $k$-colorable digraph has a coloring chip configuration modulo $k$ or $k+1$.



**Keywords:** combinatorial problems, graph algorithms, graph coloring.


## References


[1] T. Bartnicki, J. Grytczuk, S. Niwczyk, Weight choosability of graphs, J. Graph Theory (to appear).

[2] S. Czerwiński, J. Grytczuk, W. Żelazny, Lucky labelings of graphs, manuscript (2008).

[3] M. Kalkowski, M. Karoński, F. Pfender, Vertex-coloring edge-weightings: towards 1-2-3-conjecture, manuscript.

[4] M. Karoński, T. Łuczak, A. Thomason, Edge veights and vertex colours, J. Combin. Theory Ser. B 91 (2004), 151-157.

[5] J. Przybyło, M. Woźniak, 1, 2 conjecture, Preprint MD 024, available at http://www.ii.uj.edu.pl/preMD/index.php.

## DOMINATION IN UNICYCLIC GRAPHS

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Let $G = (V, E)$ be a graph without an isolated vertex. A set $D \subseteq V(G)$ is a *dominating set* if each element of $V(G) - D$ is adjacent to a vertex of $D$. A set $D \subseteq V(G)$ is a *total dominating set* if $D$ is dominating and the induced subgraph $G[D]$ does not contain an isolated vertex. The total domination number of $G$ is the minimum cardinality of a total dominating set of $G$. A set $D \subseteq V(G)$ is a *total outer-connected dominating set* if $D$ is total dominating and the induced subgraph $G[V(G) - D]$ is a connected graph. The total outer-connected domination number of $G$ is the minimum cardinality of a total outer-connected dominating set of $G$. We study total domination and total outer-connected domination in unicyclic graphs.

**Keywords:** total domination number, total outer–connected domination number, unicyclic graphs.

**AMS Subject Classification:** 05C69.

# RECENT WORK ON VIZING'S CONJECTURE

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The intriguing conjecture of Vizing concerning the domination number, $\gamma$, on the Cartesian product is that for every pair of graphs $G$ and $H$, $\gamma(G \Box H) \geq \gamma(G)\gamma(H)$. In this talk we shall survey the main results on this conjecture including the recent result of Brešar and Rall that generalizes both the early theorem of Barcalkin and German on decomposable graphs and that of Aharoni and Szabó on chordal graphs.

**Keywords:** Cartesian product, decomposable graph, 2-packing, Vizing's conjecture.

**AMS Subject Classification:** 05C69.

## STAR COLORING OF GRAPHS

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A proper coloring of the vertices of a graph is called a *star coloring* if the union of every two color classes induces a star forest. If $G$ has a proper $k$-star coloring, then $G$ is said to be $k$-star colorable. The star chromatic number $\chi_s(G)$ is the least integer $k$ such that $G$ has a $k$-star coloring. In this talk we will give a brief survey of known results concerning the star chromatic number of graphs and we will focus on some recent ones.

# ON THE RELATIONS BETWEEN LIARS' DOMINATING AND SET-SIZED DOMINATING PARAMETERS

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Assume that each vertex of a graph $G$ is the possible location for an "intruder" such as a thief, a saboteur, a fire in a facility or some possible processor fault in a multiprocessor network. Here a detection device at a vertex $v$ is assumed to be able to detect the intruder situated at any vertex in its closed neighborhood $N[v]$ ***and to identify at which vertex in*** $N[v]$ ***the intruder is located.*** Indeed, the detection device can pinpoint the location(s) of however many intruders there are in $N[v]$. The reliability problem considered here involves the situation in which a device in the neighborhood of an intruder vertex can misidentify (lie about) location(s) of the intruder(s). We define the $(i,j)$-liars' domination number of $G$, denoted by $\gamma_{LR(i,j)}(G)$, to be the minimum cardinality of a set $L \subseteq V(G)$ such that detection devices placed at the vertices in $L$ can precisely determine the set of intruder locations when there are between 1 and $i$ intruders and at most $j$ detection devices that are lying.

We also define the $\mathrm{X}(c_1, c_2, \ldots, c_t, \ldots)$-domination number, denoted by $\gamma_{\mathrm{x}(c_1,c_2,\ldots,c_t,\ldots)}(G)$, to be the minimum cardinality of a set $D \subseteq V(G)$ such that , if $S \subseteq V(G)$ with $|S| = k$, then $|(\cup_{s\in S} N[s]) \cap D| \geq c_k$. Thus, $D$ dominates each set of $k$ vertices at least $c_k$ times making $\gamma_{\mathrm{x}(c_1,c_2,\ldots,c_t,\ldots)}(G)$ a set-sized dominating parameter. We consider the relations between these set-sized dominating parameters and the liars' dominating parameters. Previously we have shown that $\gamma(G) \leq \gamma_{LR(1,1)}(G) = \gamma_{\mathrm{x}(2,3)}(G) \leq \gamma_{\times 3}(G)$. However, we have also shown that $\gamma_{LR(2,2)}(G)$ is not identical to any set-sized domination parameter. In this paper we characterize all values of $i$ and $j$ for which $\gamma_{LR(i,j)}(G)$ is equal to some $\gamma_{\mathrm{x}(c_1,c_2,\ldots,c_t,\ldots)}(G)$ parameter.

**Keywords:** liars' domination, set-sized domination, fault-tolerant reporting.
**AMS Subject Classification:** 05C69, 05C90, 94C15.

# GAME CHROMATIC NUMBER OF GRAPHS WITH LOCALLY BOUNDED NUMBER OF CYCLES

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In a coloring game Alice and Bob take turns coloring a vertex of a graph with one of $k$ colors. Alice makes the first move. Alice wins when all the vertices get colored. Bob wins if he can prevent it. Game chromatic number of graph $G$ is the smallest $k$ such that in a coloring game on graph $G$ with $k$ colors Alice has a winning strategy.

In a marking game Alice and Bob take turns marking a vertex of a graph with Alice making the first move. The score of the game is the smallest $k$ such that any unmarked vertex has less then $k$ marked neighbours during the game. Game coloring number of graph $G$ is the smallest $k$ such that Alice has a strategy guaranteeing a score of $k$. It can easily be shown that $\chi_g(G) \leq col_g(G)$.

We prove that a graph with every edge belonging to at most $c$ cycles has game coloring number of at most $c + 4$. This generalises recent result [1] that for any cactus $G$ (a graph with edge-disjoint cycles) $col_g(G) \leq 5$.

**Keywords:** game chromatic number, game coloring number.

**AMS Subject Classification:** 05C15.

## References

[1] E. Sidorowicz, The game chromatic number and the game colouring number of cactuses, Information Processing Letters 102 (2007) 147–151

# RAINBOW CONNECTION IN GRAPHS WITH MINIMUM DEGREE THREE

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An edge-coloured graph $G$ is *rainbow connected* if any two vertices are connected by a path whose edges have distinct colours. This concept of rainbow connection in graphs was recently introduced by Chartrand et al. in [2]. The *rainbow connection number* of a connected graph $G$, denoted $rc(G)$, is the smallest number of colours that are needed in order to make $G$ rainbow connected. In this talk we will show that $rc(G) < \frac{3n}{4}$ for graphs with minimum degree three, which was conjectured by Caro et al. in [1].

We will also report about the status of this problem for graphs with minimum degree at least four.

**Keywords:** Rainbow colouring, rainbow connectivity, extremal problem.

**AMS Subject Classification:** 05C35, 05C15.

## References

[1] Y. Caro, A. Lev, Y. Roditty, Z. Tuza, and R. Yuster *On rainbow connection*, The Electronic Journal of Combinatorics 15 (2008), #57.

[2] G. Chartrand, G. L. Johns, K. A. McKeon, and P. Zhang, *Rainbow connection in graphs*, Math. Bohemica, Vol. 133, No. 1, 2008, 85-98.

# ON THE CROSSING NUMBERS OF JOIN OF PATHS AND CYCLES WITH OTHER GRAPHS

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The *crossing number* $cr(G)$ of a graph $G$ is the minimum possible number of edge crossings in a drawing of $G$ in the plane. The investigation of crossing numbers of graphs is a classical and however very difficult problem. The exact values for crossing numbers are known only for few specific families of graphs.

It has been long–conjectured by Zarankiewicz [3] that the crossing number of the complete bipartite graph $K_{m,n}$ equals $\lfloor \frac{m-1}{2} \rfloor \lfloor \frac{m}{2} \rfloor \lfloor \frac{n-1}{2} \rfloor \lfloor \frac{n}{2} \rfloor$. This conjecture has been verified by Kleitman [1] for $\min\{m, n\} \leq 6$. Let $G$ and $H$ be two disjoint graphs. The *join product* of $G$ and $H$, denoted by $G + H$, is obtained from vertex–disjoint copies of $G$ and $H$ by adding all possible edges between $V(G)$ and $V(H)$. For $|V(G)| = m$ and $|V(H)| = n$, the edge set of $G + H$ is the union of disjoint edge sets of the graphs $G$, $H$, and the complete bipartite graph $K_{m,n}$.

In [2] there are established crossing numbers for join of two paths, join of two cycles, and for join of path and cycle. We extend these results and we collect the crossing numbers for join products of paths and cycles with all graphs of order four.

**Keywords:** graph, join product, drawing, crossing number, path, cycle.

**AMS Subject Classification:** 05C10.

## References

[1] Kleitman, D. J., *The crossing number of $K_{5,n}$*, J. Combin. Theory B **9** (1971), 315–323.

[2] Klešč, M., *The join of graphs and crossing numbers*, ENDM **28** (2007), 348–355.

[3] Zarankiewicz, K., *On a problem of P. Turán concerning graphs*, Fund. Math. **41** (1954), 137–145.

## DISTANCE INDEPENDENCE IN GRAPHS

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For a set $D$ of positive integers, we define a vertex set $S \subseteq V(G)$ to be $D$-independent if $u, v \in S$ implies the distance $d(u, v) \notin D$. The $D$-independence number $\beta_D(G)$ is the maximum cardinality of a $D$-independence set. In particular, the independence number $\beta(G) = \beta_{\{1\}}(G)$. Here we present results about this general parameter and its relations to other distance parameters.

**Keywords:** independence number, domination number, distance set.

# INDEPENDENT SETS MEETING ALL LONGEST PATHS

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Laborde, Payan and Xuong [1] conjectured that every digraph has an independent set of vertices that meets every longest path. We consider the conjecture for special classes of oriented graphs.

**Keywords:** Oriented graphs, longest paths, independent sets.

**AMS Subject Classification:** 05C20, 05C38, 05C69.

## References

[1] J.M. Laborde, C. Payan, and N.H. Xuong, Independent sets and longest directed paths in digraphs, in: *Graphs and other combinatorial topics* (Prague, 1982) 173–177 (Teubner-Texte Math., 59, 1983.)

# STRUCTURE OF GRAPHS WITH NUMEROUS DOMINATING SETS

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Efficiently dominating, minimally dominating or all dominating sets are dealt with. The structure of some graphs with extremal numbers of such sets is the subject of the talk.

**AMS Subject Classification:** 05C69, 05C05, 05C35, 05C75, 05A15.

## References

[1] Dorota Bród, Z. Skupień, Trees with extremal numbers of dominating sets, Australas. J. Combin. 35 (2006) 273–290.

[2] D. Bród, Z. Skupień, Recurrence among trees with most numerous efficient dominating sets, Discrete Math. Theoret. Comp. Sci. 10:1 (2008) 43–56.

[3] Anna Kędzior, Z. Skupień, A. Żak, On the structure of graphs with most numerous minimal dominating sets, manuscript.

# COMPLEMENTARITY AND DUALITY OF GENERALIZED GRAPHICAL SUBSET PROBLEMS

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Gallai's Theorem states that the minimum cardinality of a cover, denoted $\alpha(G)$, and the maximum cardinality of an independent set, denoted $\beta(G)$, satisfy $\alpha(G)+\beta(G) = |V(G)|$. There are many other such set-complementation theorems involving set minimization and set maximization parameters. For example, the domination and enclaveless parameters satisfy $\gamma(G) + \Psi(G) = |V(G)|$. Extensions from subset problems involving $f : V(G) \to \{0,1\}$ or $f : E(G) \to \{0,1\}$ to generalized parameters that are Y-valued for an arbitrary subset Y of the reals produce more general complementation theorems. The Matrix Complementation Theorem will be presented.

Likewise, graph theoretic minimization (respectively, maximization) problems expressed as linear programming problems have dual maximization (respectively, minimization) problems. The LP-duality Theorem relates the values of these parameters. Again, one can generalize to an arbitrary set Y of reals. A particular case with $Y = \{0,1\}$ is that domination and packing are dual problems.

The relations among parameters arising by (successively) taking duals and complements will be examined.

**Keywords:** dominating, enclaveless, packing, independent, covering, edge cover.

**AMS Subject Classification:** 05C69, 05C15.

# LARGE VERTEX-TRANSITIVE AND CAYLEY DIGRAPHS WITH GIVEN DEGREE AND DIAMETER

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The degree-diameter problem is the problem of finding largest possible graphs and digraphs of given degree and diameter. Usually, symmetric graphs and digraphs are in the center of interest. The motivation for studying large vertex-transitive (and Cayley) digraphs with given degree and diameter comes from potential applications in symmetric interconnection networks. One of the known constructions of large vertex-transitive digraphs is due to Faber, Moore and Chen [1,2].

In our contribution we determine the automorphism group of the so called Faber-Moore-Chen digraphs and we establish necessary and sufficient conditions for a Faber-Moore-Chen digraph to be a Cayley digraph.

**Keywords:** digraph, degree-diameter problem, Cayley digraph.

## REFERENCES

[1] V. Faber and J. W. Moore, High-degree low-diameter interconnection networks with vertex symmetry: the directed case, Technical Report LA-UR-88-1051, Los Alamos National Laboratory, Los Amalos, NM (1988).

[2] V.Faber, J.W.Moore and W.Y.C. Chen, Cycle prefix digraphs for symmetric interconnection networks, Networks 23 (1993) 641-649.

# COUNTING MAXIMAL INDEPENDENT SETS IN GRAPHS WITH MAXIMUM DEGREE 3

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Moon and Moser proved that the number of maximal independent sets in a graph on $n$ vertices is bounded by $3^{n/3}$. There are some algorithms that generate all maximal independent sets with time proportional to their number. Dahllöf, Jonsson and Wahlström gave an algorithm that counts the number of all independent sets (not necessary maximal) in time $O^*(1.25^n)$, what is faster than generating.

We give an algorithm which counts all maximal independent sets of a graph with maximal degree three in time $O^*(1.26\ldots^n)$.

**Keywords:** maximal indepentent sets, counting, algorithms.

**AMS Subject Clssification:** primary 05C85, secondary 05C69.

## REFERENCES

[1] J. Moon, L. Moser *On cliques in graphs*

[2] V. Dahllöf, P. Jonsson, M. Wahlström *Counting models for $2SAT$ and $3SAT$ formulae*

[3] M. Fürer and S. Kasiviswanathan *Algorithms for counting $2SAT$ solutions and colorings with applications*

# NEW TYPES OF PROBLEMS ON HYPERGRAPH COLORING

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Let $\mathcal{H} = (V, \mathcal{E})$ be a hypergraph with vertex set $V$ and edge set $\mathcal{E}$; that is, $\mathcal{E}$ is a set system over $V$. Our general approach to vertex coloring is to define four functions $s, t, a, b : \mathcal{E} \to \mathbb{N}$, with $1 \le s(E) \le t(E) \le |E|$ and $1 \le a(E) \le b(E) \le |E|$ for all $E \in \mathcal{E}$, with the following meaning. A coloring $\varphi : V \to \mathbb{N}$ is considered to be proper if, for all $E \in \mathcal{E}$,

- there are at least $s(E)$ distinct colors on $E$,
- there are at most $t(E)$ distinct colors on $E$,
- some color occurs at least $a(E)$ times on $E$,
- every color occurs at most $b(E)$ times on $E$.

A structure with all these conditions is called a *stably bounded hypergraph*; and if only $s$ and $t$ can be restrictive functions, then it is called a *color-bounded hypergraph.*

A subclass of very high importance is that of *mixed hypergraph*, in which each edge $E$ is required to contain two vertices with a common color (*C-edge*, corresponding to $s(E) = 1$, $t(E) = |E| - 1$, $a(E) = 2$, $b(E) = |E|$) or two vertices with distinct colors (*D-edge*, with $s(E) = 2$, $t(E) = |E|$, $a(E) = 1$, $b(E) = |E| - 1$), or both, i.e. a monochromatic pair and also a 2-colored pair has to occur in it (*bi-edge*, with $s(E) = a(E) = 2$, $t(E) = b(E) = |E| - 1$).

The classical theory of (hyper)graph coloring assumes that no monochromatic edges occur; this means a (hyper)graph with D-edges only. The other side, when completely multicolored edges are forbidden, is called *C-coloring* or *C-hypergraph* (depending on context). This simple and natural condition also leads to very interesting questions.

## ON FULKERSON CONJECTURE

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If $G$ is a bridgeless cubic graph, Fulkerson conjectured that we can find 6 perfect matchings (a *Fulkerson covering*) with the property that every edge of $G$ is contained in exactly two of them. A consequence of the Fulkerson conjecture would be that every bridgeless cubic graph has 3 perfect matchings with empty intersection (this problem is known as the Fan Raspaud Conjecture). A *FR-triple* is a set of 3 such perfect matchings. We show here how to derive a Fulkerson covering from two FR-triples.

Moreover, we give a simple proof that the Fulkerson conjecture holds true for some classes of well known snarks.

**Keywords:** Cubic graph; Perfect Matchings.

**AMS Subject Classification:** 05C70.

### References

[1] G. Fan and A. Raspaud, *Fulkerson's conjecture and circuit covers*, J. Comb. Theory Ser. B **61** (1994), 133–138.

[2] J.L. Fouquet and J.M. Vanherpe, *On parsimonious edge-colouring of graphs with maximum degree three*, Tech. report, LIFO, april 2009.

[3] D.R. Fulkerson, *Blocking and anti-blocking pairs of polyhedra*, Math. Programming **1** (1971), no. 69, 168–194.

[4] M.K. Goldberg, *Construction of class 2 graphs with maximum vertex degree 3*, J. Combin. Theory B **31** (1981), 282–291.

[5] R. Isaacs, *Infinite families of non-trivial trivalent graphs which are not Tait colorable*, Am. Math. Monthly **82** (1975), 221–239.

[6] R. Rizzi, *Indecomposable $r$-graphs and some other counterexamples*, Journal of Graph Theory **32** (1999), no. 1, 1–15.

[7] J.J. Watkins, *Snarks*, Graph theory and its applications: East and West (Jinan,1986), Ann. New York Acad. Sci., vol. 576, New York Acad. Sci., 1989, New York, pp. 606–622.

[8] Rongxia Hao, Jianbing Niu, Xiaofeng Wang, Cun-Quan Zhang and Taoye Zhang, *A note on Berge-Fulkerson coloring*, Discrete Mathematics (2009).

# LIST COLORINGS OF GRAPHS

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Let $G$ be a simple graph. A ***list assignment*** $L$ of $G$ is a function that assigns to every vertex $v$ of $G$ a set (list) $L(v)$ of colors. We say that $G$ is ***$L$-colorable*** if the vertices of $G$ can be properly colored (i.e. adjacent vertices receive distinct colors) so that each vertex is colored by a color from its list. The list assignment $L$ is called a ***$k$-assignment*** if $|L(v)| = k$ for all $v \in V$. The graph $G$ is ***$k$-list colorable*** if it is $L$-colorable for every $k$-assignment $L$. The ***list chromatic number*** $\chi_\ell(G)$ is the minimum integer $k$ such that $G$ is $k$-list colorable.

The talk summarizes some results and open problems in this very rich field of research including the following topics.

**1. Minimal $k$-list critical graphs**

A graph $G$ is *$L$-critical* for a given list assignment $L$ if every proper subgraph of $G$ is $L$-colorable, but $G$ itself is not $L$-colorable. $G$ is called ***$k$-list critical*** if there is a ($k$-1)-assignment $L$ such that $G$ is $L$-critical. If $G$ is $k$-list critical and $G$ does not contains a $k$-list critical graph as a proper subgraph then $G$ is *minimal $k$-list critical.* Note that a graph $G$ is minimal $k$-list critical if and only if $\chi_\ell(H) < \chi_\ell(G) = k$ for every proper subgraph $H$ of $G$.

**2. Precoloring extension**

Let $G = (V, E)$ be a graph and $L$ a list assignment with special properties, e.g. $|L(v)| = \Delta(G)\ \forall v \in V$ or $|L(v)| = 4\ \forall v \in V$ if $G$ is outerplanar. Moreover let $W \subseteq V$ be a subset of $V$ such that $G[W]$ is $s$-colorable and assume that each component of $G[W]$ is properly precolored by $s$ colors. Denote the shortest distance between components of $G[W]$ by $d(W)$. We are interested in bounds for $d(W)$ such that any such precoloring extends to an $L$-coloring of $V$?

**3. List colorings for special list assignments**

We look for questions like the following asked by Joan Hutchinson and Carsten Thomassen, respectively.

Let $G$ be a planar, 3-connected graph which is not a complete graph. Is there an integer $k$ such that $G$ is $L$-colorable for every list assignment $L$ with $|L(v)| = \min\{d(v), k\}$ for all $v \in V$?

## GENERALIZED PELL NUMBERS AND THEIR GRAPH REPRESENTATIONS

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Let k be a fixed integer, $k \geq 2$. A subset $S \subset V(G)$ is a k-independent set of G if for each two distinct vertices $x, y \in S$, $d_G(x, y) \geq k$.

We propose a generalization of the Fibonacci numbers, the Lucas numbers, the Pell numbers, the Pell-Lucas numbers, the Tribonacci numbers and next we give their graph representations with respect to the number of k-independent sets in special graphs.

### REFERENCES

[1] M. Kwaśnik, I. Włoch, *The total number of generalized stable sets and kernels of graphs*, Ars Combin., 55 (2000), 139-146.

[2] I. Włoch, *On generalized Pell numbers in graphs*, Comment. Math. 48 (2), (2008), 169-175.

# CYCLIC PARTITIONS OF COMPLETE UNIFORM HYPERGRAPHS

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(joint work with Artur Szymański)

By $K_n^{(k)}$ we denote the complete $k$-uniform hypergraph of order $n$, $1 \leq k \leq n-1$, i.e. the hypergraph with the set $V_n = \{1, 2, ..., n\}$ of vertices and the set $\binom{V_n}{k}$ of edges. If there exists a permutation $\sigma$ of the set $V_n$ such that $\{E, \sigma(E), ..., \sigma^{p-1}(E)\}$ is a partition of the set $\binom{V_n}{k}$ then we call it $p$-cyclic partition of $K_n^{(k)}$ and $\sigma$ a permutation of $p$-cyclic partition of $K_n^{(k)}$. If $p = 2$ we have $\sigma(E) = \binom{V_n}{2} - E$ and the hypergraph $(V_n; E)$, and $(V_n; \sigma(E))$ as well, is self-complementary $k$-uniform hypergraph.

We shall give several necessary and sufficient conditions for a permutation to be a permutation of $p$-cyclic partition of $K_n^{(k)}$.

It is true that that if $\binom{n}{k}$ is even then there is a $k$-uniform self-complementary hypergraph of order $n$. The corresponding result is no longer true for $p$-cyclic partitions of $K_n^{(k)}$, that is there are such $p, k$ and $n$ that $\binom{n}{k}$ is divisible by $p$, but there is no $p$-cyclic partition of $K_n^{(k)}$. We shell discuss the problem for which $p, k$ and $n$ there is a $p$-cyclic partition of $K_n^{(k)}$? The problem of $p$-cyclic partitions of the complete hypergraph $(V_n; 2^{V_n} - \{V_n, \emptyset\})$ will be also discussed and solved.

# THE EDGE SPAN OF GRAPH DISTANCE LABELING

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The distance labeling (or coloring) of a graph is an assignment of numbers (or labels) to the vertices with conditions depend on the distance between vertices. This class of graph labeling is motivated by the frequency assignment problem.There are considerable efforts on this labeling since it was introduced in 1992.

Given a graph $G = (V, E)$ and nonnegative integers $k_1 \geq k_2 \geq k_3$, an $L(k_1, k_2, k_3)$-*labeling* of $G$ is an assignment $f : V \to \{0, 1, \ldots\}$ such that $|f(u) - f(v)| \geq k_i$ whenever the distance between $u$ and $v$ is $i$ in $G$, for $i = 1, 2, 3$. The tuple $(k_1, k_2, k_3)$ is called the *constraint* of the labeling. The $L(k_1, k_2, k_3)$-*span* is the smallest number $m$ such that there is an $L(k_1, k_2, k_3)$-labeling with the maximum value $m$. If $k_3 = 0$ then the constraint is denoted by $(k_1, k_2)$ for short. In the previous study, people are interested in finding $L(k_1, k_2)$-spans with various $k_1$ and $k_2$. Given an $L(k_1, k_2, k_3)$-labeling $f$ of a graph $G$, the *edge span* of $f$ is defined by $\max\{|f(u) - f(v)| : uv \in E(G)\}$. The $L(k_1, k_2, k_3)$-*edge span* of $G$ is the minimum edge span over all $L(k_1, k_2, k_3)$-labelings of $G$ and is denoted by $\beta(G; k_1, k_2, k_3)$.

In a communication network, large service areas are often covered by a network of congruent polygonal cells with each station or transmitter at the center of cell that it covers. There are only three regular tilings (regular cell coverings ) can cover the whole plane, which are square tiling, hexagonal tiling and triangular tiling. Correspondingly, we have the square lattice, the triangular lattice and the hexagonal lattice.

Since our labeling problem was motivated by the channel assignment problem of a communication network,this talk will present recent results on edge spans with constraints $(k_1, k_2, k_3)$ for these three classes of graphs.

**Keywords:** graph coloring, distance.

**AMS Subject Classification:** 05C78.

# ON GEODETIC SETS OF A GRAPH*

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In a graph $\Gamma = (V, E)$, a $u-v$-geodesic between the vertices $u, v \in V$ is formed by the vertices $u, v$ and the all vertices lying in a path of minimum length between these two vertices. A subset of vertices $S \subset V$ is a geodetic set [1, 2], if every vertex of the graph lies on some geodesic between two vertices $u, v \in S$. The geodetic number of the graph is the minimum cardinality of any geodetic set and it is denoted by $g(\Gamma)$.

A subset $S \subset V$ is a $k$-geodetic set [3], if all the vertices in the graph lies on some geodesic of length $k$ between two vertices $u, v \in S$. The minimum cardinality of any $k$-geodetic set is called the $k$-geodetic number of the graph and it is denoted by $g_k(\Gamma)$. Also, the subset $S \subset V$ is a total $k$-geodetic set if every vertex $v$ of the graph lies in a $u_1 - u_2$-geodesic of length $k$ between two vertices $u_1, u_2 \in S$, with $v \neq u_1$ and $v \neq u_2$. In this case, the total $k$-geodetic number of the graph is the minimum cardinality of any total $k$-geodetic set and it is denoted by $g_k^t(\Gamma)$.

Here, we study the geodetic and $k$-geodetic numbers of the cartesian product of graphs. For instance we obtain that, for any connected graph $\Gamma_i$ of order $n_i$, $i \in \{1, 2\}$, $g(\Gamma_1 \times \Gamma_2) \leq n_1 + n_2 - 2$. Also we present the relationship between the total $k$-geodetic number of two graphs $\Gamma_1$ and $\Gamma_2$ and the $k$-geodetic number of the cartesian product $\Gamma_1 \times \Gamma_2$ of these two graphs, so we have obtained that $g_{k_i+k_2}(\Gamma_1 \times \Gamma_2) \leq g_{k_1}^t(\Gamma_1) g_{k_2}^t(\Gamma_2) - \min\{g_{k_1}^t(\Gamma_1), g_{k_2}^t(\Gamma_2)\}$.

We also study the relationship between dominating sets, independent sets and geodetic and $k$-geodetic sets and we obtain some tight bounds for the geodetic and $k$-geodetic numbers related to the diameter, the order and the minimum and maximum degree of the graph.

**Keywords:** Geodetic sets, dominating sets, cartesian product.

**AMS Subject Classification:** 05C69; 05C70.

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## References


[1] G. Chartrand, F. Harary y P. Zhang, On the geodetic number of a graph, *Networks* **92**, (3) (1985), 211-212.

[2] F. Harary, E. Loukakis and C. Tsouros, The geodetic number of a graph, *Math. Comput. Modelling* **17** (11), (1993) 89-95.

[3] R. Muntean and P. Zhang, $k$-geodomination in graphs, *Ars Combinatoria*, **63** (2002), 33-47.

# FIXED SIMPLEX PROPERTY FOR RETRACTABLE COMPLEXES

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Retractable complexes are defined by authors. It is proved that every simplicial map defined on the set of all vertices of a retractable complex into itself has a fixed simplex. This generalise a well-known Rival-Nowakowski theorem for trees: every simplicial map transforming vertices of a tree into itself has a fixed edge [2] and Hell and Nešetřil theorem: any endomorphism of a dismantible graph fixes some clique [1].

**Keywords:** fixed simplex, retractable $\leq$n- complex, retraction, simplicial map.

**AMS Subject Classification:** Primary 05B30, 47H10; Secondary 52A20, 54H25.

## References

[1] P. Hell and J. Nešetřil: *Graphs and Homomorphisms*, Oxford LS in M., 2004

[2] R. Nowakowski and I. Rival: *Fixed-Edge Theorem for Graphs with Loops*, Journal of Graph Theory 3 (1979) 339-350

## 2 AND 3-EXISTENTIALLY CLOSED TOURNAMENTS

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A tournament $T$ is $k-$existentially closed ($k \geq 1$), if for every subset $A \subseteq V(T)$ such that $\mid A \mid= k$ and every $B \subseteq A$, there exists $x \notin A$ such that $x$ dominates every element of $B$ and every element of $A \setminus B$ dominates $x$. We will say that a $k-$existentially closed tournament $T$ has property $P_k$. In this talk we present families of tournaments with property $P_2$ and $P_3$.

**Keywords:** $k-$existentially closed tournament, circulant tournamet, Paley tournament, Szekeres tournament.

**AMS Subject Classification:** 05C20.

### REFERENCES

[1] Bonato Anthony, Cameron Kathie. On an adjacency property of almost all tournaments. Discrete Math. 306 (2006), no. 19-20, 2327–2335.

[2] Llano Bernardo, Zuazua Rita. 2 and 3-existentially closed tournaments (submitted).

# PACKING GRAPHS WITHOUT SHORT CYCLES IN THEIR COMPLEMENTS

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The following statement was conjectured by Faudree, Rousseau, Schelp and Schuster [2].
*If a graph $G$ is a non-star graph without cycles of length $m \leq 4$ then $G$ is a subgraph of its complement.*

So far the best result concerning this conjecture is that every non-star graph $G$ without cycles of length $m \leq 6$ is a subgraph of its complement. This was proved by Brandt [1]. Another, relatively short proof of Brandt's result was given by Görlich, Pilśniak, Woźniak and Zioło [3]. In the talk we will show that $m \leq 6$ can be replaced by $m \leq 5$.

**Keywords:** packing, graph complement, short cycles

## References

[1] S. Brandt, *Embedding graphs without short cycles in their complements*, in: Y. Alavi, A. Schwenk (Eds.), Graph Theory, Combinatorics, and Application of Graphs, 1 (1995) 115–121

[2] R.J. Faudree, C.C. Rousseau, R.H. Schelp, S. Schuster, *Embedding graphs in their complements*, Czechoslovak Math. J. 31 (106) (1981) 53–62

[3] A. Görlich, M. Pilśniak, M. Woźniak, I. A. Zioło, *A note on embedding graphs without short cycles*, Discrete Math. 286 (2004) 75–77.

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