12th WORKSHOP ON GRAPH THEORY

COLOURINGS, INDEPENDENCE AND DOMINATION

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ABSTRACTS

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Abstracts

ARC-TRANSITIVE CAYLEY GRAPHS ON METACYCLIC p-GROUPS

MEHDI ALAEIYAN AND MOHSEN GHASEMI

For a group G, and a subset S of G such that $1_G \notin S$, let $\Gamma = Cay(G, S)$ be a Cayley graph. Then Γ is said to be arc-transitive, if $Aut(\Gamma)$ act transitive on the set of its arcs. In this paper we classify arc-trnsitive Cayley graphs on metacyclic groups of order p^3.

Keywords: Cayley graph, normal Cayley graph, arc-transitive Cayley graph.

DOMINATION IN GRAPHS AND MULTILINEAR FUNCTIONS

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A multilinear function f in n variables is defined and it is proved that the domination number $\gamma(G)$ of a graph G with $|V(G)| = n$ equals the minimum of f taken over the n-dimensional cube $[0,1]^n$. Discussing this continuous optimization problem new uper bounds on $\gamma(G)$ are established.

AN ALGORITHM FOR GENERATING GRAPHS WITH A GIVEN CHROMATIC NUMBER

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A simple graph on n vertices with vertex degree bounded by $f \leq n-2$ is called an f-graph. An edge maximal f-graph is a graph to which no edge can be added without violating the f-degree restriction [1].

Random edge maximal f-graphs can be generated as outcomes of a process called the *Random f-Graph Process* (RfGP) whose states are f-graphs [2]. The initial state of the RfGP is the empty graph of order n. One step of the process is adding one edge to an f-graph G and obtaining a new state. The edge to be added is *selected uniformly* from the set of all *admissible edges*, i.e. edges of the complement of G, which can be added to G without introducing a vertex of degree greater than f. The process stops when the set of admissible edges is empty. A terminal state of the RfGP is an edge maximal f-graph.

Chromatic properties of edge maximal f-graphs of order n generated by the RfGP have been studied using methods of graph theory and algorithms, both exact and randomized. Conditions for generating graphs with a given chromatic number are defined.

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MULTICOLOR RAMSEY NUMBERS FOR SOME GRAPHS

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We give the multicolor Ramsey number for some graphs with a complete graph or a cycle in the given sequence generalizing a results of Stahl [3], Baskoro *et al.* [1] and Dzido [2].

Keywords: complete graph, cycle, forest, G-good graph, Ramsey number, tree.

AMS Subject Classification: 05C55.

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SOME RESULTS IN A CONSENSUS LIST COLOURING

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Let $S(v)$ denote a nonempty set of integers assigned to vertex v of graph $G = (V, E)$. Let call S a list assignment for G. We seek a proper graph colouring f such that for every vertex $v \in V$ we have $f(v) \in S(v)$. Such a colouring is called a list colouring for (G, S). We consider one of consensus models described in [2] called trading model. In situation when such colouring can not be obtained in trading model there is a possibility to create new list assignment S' based on S. A new assignment S' is created in a series of steps called trades. A trade from a vertex u to v means that we remove colour c from $S(u)$ and add it to $S(v)$.

We ask how many trades are required in order to obtain a list assignment that has a list colouring. We say that (G, S) is p-tradeable if this can be done in p trades.

We show a polynomial algorithm based on maximum cardinality matching in bipartite graphs to determine minimal p for given (G, S) where G is a complete graph. We also prove that determine if (G, S) is p-tradeable for given p where G is a tree is an NP-complete problem.

Keywords: list colouring, consensus list colouring, trading model.

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MATCHING CUTSETS IN GRAPHS OF DIAMETER 2

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A matching cutset of a graph $G = (V, E)$ is a set $M \subseteq E$ of independent edges such that $G - M$ has more components than G. Chvátal [1] proved that recognizing graphs with a matching cutset is NP-complete. Since then the graph classes for which this problem is polynomial were investigated. Interesting results were presented by Moshi in [4]. He proved that the matching cutset problem is solvable in polynomial time when we restrict the input graphs to line graphs or graphs without induced cycles of lenght ≥ 5. In the same paper [4] he also proved that the problem remains NP-complete for bipartite graphs of minimum degree 2.

In this talk we present a polynomial-time algorithm which solves the problem of recognizing graphs with a matching cutset for graphs of diameter two. We say that a graph possess a module if there exists a set $W \subset V$ such that every vertex outside W is either adjacent to all the vertices of W or to none of them. The notion of the module plays an important role in our algorithm.

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ON THE DECOMPOSITION OF GRAPHS

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We denote by $\mathcal{I}$ the class of all finite simple graphs. A ***graph property*** is a nonempty isomorphism-closed subclass of $\mathcal{I}$. A property $\mathcal{P}$ is called ***hereditary*** if it is closed under subgraphs.

Let $\mathcal{P}_1, \mathcal{P}_2, \ldots, \mathcal{P}_n$ be hereditary properties. A $(\mathcal{P}_1, \mathcal{P}_2, \ldots, \mathcal{P}_n)$-***decomposition*** of a graph $G \in \mathcal{I}$ is a partition $E_1, E_2, \ldots, E_n$ of $E(G)$ such that the subgraph induced by E_i has the property $\mathcal{P}_i$, for $i = 1, 2, \ldots, n$. We denote by $\mathcal{P}_1 \oplus \mathcal{P}_2 \oplus \cdots \oplus \mathcal{P}_n$ the property $\{G \in \mathcal{I} : G$ has a $(\mathcal{P}_1, \mathcal{P}_2, \ldots, \mathcal{P}_n)$-decomposition$\}$. A property $\mathcal{P}$ is said to be ***decomposable*** if there exist nontrivial hereditary properties $\mathcal{P}_1$ and $\mathcal{P}_2$ such that $\mathcal{P} = \mathcal{P}_1 \oplus \mathcal{P}_2$. We study the relations between decomposable properties. We shall deal with the following hereditary properties:

$\mathcal{O}_1 = \{G \in \mathcal{I}$: each component of G has at most two vertices$\}$,
$\mathcal{D}_k = \{G \in \mathcal{I} : G$ is k-degenerate,
i.e., every subgraph of G has a vertex of degree at most $k\}$,
$\mathcal{D}_1^* = \{G \in \mathcal{I} : G$ contains at most one cycle$\}$.
We prove that $\mathcal{D}_1 \oplus \mathcal{D}_1^* \subseteq \mathcal{O}_1 \oplus \mathcal{D}_2$.

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ACYCLIC COLOURINGS OF GRAPHS

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For a given graph $G = (V, E)$ and a sequence $\mathcal{P}_1, \mathcal{P}_2, \ldots, \mathcal{P}_k$ of additive hereditary properties of graphs we define a $(\mathcal{P}_1, \mathcal{P}_2, \ldots, \mathcal{P}_k)$-*colouring* of G as a partition $(V_1, \ldots, V_k)$ of V such that $G[V_i] \in \mathcal{P}_i$, $i = 1, \ldots, k$. Such a colouring is called *acyclic* if for every two distinct colours i and j, the subgraph induced by all the edges linking an i-coloured vertex and a j-coloured vertex does not contain a cycle.

A property $\mathcal{R}$ consisting of all graphs having an acyclic $(\mathcal{P}_1, \mathcal{P}_2, \ldots, \mathcal{P}_k)$-colouring will be called *an acyclic reducible property.* We consider acyclic reducible properties which are upper bounds for some classes of graphs in the lattice of all additive hereditary properties.

Keywords: acyclic colouring, additive hereditary property.

AMS Subject Classification: 05C15.

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NEW 3-COMPETITIVE ALGORITHM FOR ON-LINE COLORING OF INTERVAL GRAPHS

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Within this talk we present algorithm `IC` (Interval Coloring) - new on-line algorithm for coloring of interval graphs. Algorithm `IC` achieves the best possible worst case performance ratio [4] and in average case it compares favourably with other on-line 3-competitive algorithms given in [4, 5].

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ON THE 2-RAINBOW DOMINATION IN GRAPHS

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The concept of 2-rainbow domination of a graph G coincides with the ordinary domination of the prism $G\Box K_2$. In this talk we show that the problem of deciding if a graph has a 2-rainbow dominating function of a given weight is NP-complete even when restricted to bipartite graphs or chordal graphs. Exact values of 2-rainbow domination numbers of several classes of graphs are found, and it is shown that for the generalized Petersen graphs $GP(n,k)$ this number is between $\lceil 4n/5 \rceil$ and n with both bounds being sharp.

Keywords: complexity, algorithm, NP-completeness, domination, Cartesian product, generalized Petersen graph.

AMS Subject Classification: 05C85, 05C69.

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MINIMUM CONNECTED DOMINATING SETS IN UNIT DISK GRAPHS

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A unit disk graph is a graph having points in the Euclidean plane as vertices and any two vertices are joined by an edge if the distance between them is at most 1. Finding the minimum connected dominating set in unit disk graphs is NP-hard problem which plays an important role in efficient routing in ad hoc wireless networks. Many approximation algorithms (see, for example, [2, 3]) construct maximal independent set at first step. The relation between the size $mis(G)$ of a maximum independent set and the size $cds(G)$ of a minimum connected dominating set in the same graph G is used to determine the performance ratio of such algorithms. It is shown in [1] that in every unit disk graph G, $mis(G) \leq 3.8 \cdot cds(G) + 1.2$. We improve this result by showing that $mis(G) \leq 3.6 \cdot cds(G) + 1.4$.

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HAMILTONICITY OF k-TRACEABLE GRAPHS

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A graph is called k-*traceable* if each of its induced subgraphs of order k is traceable. It follows easily from Dirac's degree condition for hamiltonicity that for $k \geq 2$ every k-traceable graph of order $n \geq 2k - 1$ is hamiltonian. We suspect that the bound on n may be reduced considerably. For each $k \geq 2$ we define $H(k)$ to be the largest integer such that there exists a k-traceable graph of order $H(k)$ that is nonhamiltonian. We determine the exact value of $H(k)$ for $k \leq 8$ and show that $k + 2 \leq H(k) \leq \frac{3k-5}{2}$ for $k \geq 10$.

STRUCTURAL ANALYSIS OF GRAPHS BY USING INFORMATION THEORETIC FUNCTIONALS

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1. Topic and Results

The problem to measure the structural information content of graphs (graph entropy) has been frequently investigated, see e.g., [1, 3, 4, 5, 7]. Most of these classical methods to determine graph entropy are based on the problem to find a partition of the underlying vertex set [4, 5, 7] which can be very difficult for arbitrary graphs. In contrast to this, we give first a novel definition of graph entropy by avoiding the problem of determining certain vertex partitions. We define functionals which are based on metrical and eccentrical graph properties [6] to define a vertex probability that finally leads us to a graph entropy. Based on this approach we state some lower and upper bounds for the defined graph entropy. Additionally, based on a novel graph similarity measure we compare graphs structurally by using the relative entropy that is also called Kullback-Leibler divergence [2].

Keywords: graphs, metrical and eccentrical quantities, entropy, graph similarity.

AMS Subject Classification: 05C99.

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COMPUTING THE NUMBER OF INDEPENDENT SETS USING FIBONACCI RELATIONS

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Counting the number of independent sets of a graph G, denoted by $NI(G)$, is a classical #P-complete problem for graphs of degree 3 or higher [1]. We exploit the relation between $NI(G)$ and the Fibonacci numbers that allow us to establish new polynomial classes for $NI(G)$ problem.

Such new classes are determined based on the topological structure of the graph. We show that if the depth-first search over a graph G generates a graph where the set of fundamental cycles are non-intersected, that is, there are not common edges between any pairs of fundamental cycles (although they could share nodes) then, $NI(G)$ is computed in polynomial time, in fact, in linear time with respect to the length of the input graph G, that is, our proposal has a time complexity of $O(n+m)$, n and m being the number of nodes and edges in the graph, respectively. We have called to this class of graphs, *the class of topological ordered graphs.*

This new polynomial class is a superclass of graphs of degree 2 and it has not restrictions over the degree of the graphs, but rather it depends on the topological structure of the graphs. The class of topological ordered graphs allows us to establish a finer border between the classes FP and #P for counting the number of independent sets.

This topological approach allows us to identify a common structure for all tractable graphs where to count its combinatorial objects, such as counting models in Boolean Formulas, counting colourings of nodes, counting edge coverings, etc.. they can be computed in polynomial time.

Our algorithm could be adapted for obtaining faster algorithms for counting the number of independent sets for graphs with any degree, i.e. we show a leading Worst-case upper bound of $O(poly(n) * \phi^{(n+1)/2})$ for computing $NI(G)$, where n is the number of nodes and $\phi = 1/2(1+\sqrt{5})$ is the 'golden ratio'.

Keywords: counting the number of independent sets, enumerative combinatory.

AMS Subject Classification: 05A06.

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NEW POLYNOMIAL CLASSES FOR COUNTING THE NUMBER OF 3-COLORINGS OF A GRAPH

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Counting problems in graph theory that can be solved exactly in polynomial time are few. In this work, we design different efficient algorithms to carry out the exact counting of the number of 3-colorings for any undirected graph.

The problem #3-Col(G) which counts the number of 3-colorings of a graph G is a classic #P-complete problem for graphs G of degree 3 or higher [1]. Contrary to this latter result, we establish new polynomial classes of graphs where to compute #3-Col(G) is done in linear time without restrictions on the degree of the graph, but rather it depends on the topological structure of the graph. We show that #3-Col(G) can be computed in linear time if G has the following topological structure:

1. G is a tree (this case includes when G is a chain graph).
2. G does not have any intersected cycles (this case includes when G is a simple cycle).
3. If G has a set of intersected cycles, such set can be translated, via a polynomial time reduction, to a set of embedded cycles with a common start node.

For all those cases, we have developed linear-time algorithms based on traversing G in depth-first search. We associate to each node v of the graph an ordered triple: $(\alpha_v, \beta_v, \gamma_v)$ of integer numbers which carries the number of times that the node v could be colored with the first, second and third color, respectively. According to the depth-first search when a new node $w \in V(G)$ adjacent to v is visited the first time, we compute the new ordered triple $(\alpha_w, \beta_w, \gamma_w)$ in base on the values of $(\alpha_v, \beta_v, \gamma_v)$ and according if the edge $\{v, w\}$ is a back edge or a tree edge in the depth-first graph.

The exact method showed here could be used to impact directly to speed up many other algorithms for counting problems, i.e. for counting models in propositional Boolean formulas, counting the number of independent sets, counting exact covers, etc.

Keywords: counting colorings of a graph, exact counting, enumerative combinatory.

AMS Subject Classification: 05A06.

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ON MONOCHROMATIC PATHS AND BICOLORED SUBDIGRAPHS IN ARC-COLORED TOURNAMENTS

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Let D be a digraph. D is said to be an m-colored digraph if the arcs of D are colored with m colors. A path P in D is called a monochromatic one if all of its arcs are colored alike. Now, let D be an m-colored digraph. A set $N \subseteq V(D)$ is said to be a **kernel by monochromatic paths** of D if for every pair of different vertices u and v in N there is no monochromatic directed path in D between them and for every vertex $x \in V(D) - N$ there is a vertex $n \in N$ such that there is a xn-monochromatic directed path in D.

This concept is a generalization of a kernel. We prove new and different sufficient conditions which imply that arc-colored tournaments have kernel by monochromatic paths. Our conditions concern to some subdigraphs of tournaments and their almost monochromatic and bicolor coloration too. We also prove that our conditions are not mutually implied. At the end some open problems are proposed.

Keywords: kernel, kernel by monochromatic paths, tournaments.

AMS Subject Classification: 05C20, 05C38, 05C69.

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GENERALIZATIONS OF THE GRAPH RANKING PROBLEM

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Given a simple graph $G = (V, E)$, a function c mapping the set of vertices of G into integers $1, \ldots, k$ is a vertex k-ranking of G if each path connecting two vertices of the same color contains a vertex with a bigger color. The smallest number k such that the above function c does exist is called a vertex ranking number of G and denoted by $\chi_r(G)$. If we label the edges of the graph instead of its vertices then the corresponding function c is an edge k-ranking of G and the smallest number k is called in this case an edge ranking number of G.

In this talk we survey some generalizations and modifications of the above problems, including some new results. We discuss algorithmic aspects of the selected graph ranking problems. We are especially interested in determining computationally easy and hard instances.

MINIMAL FORBIDDEN SUBGRAPHS OF H-REDUCIBLE GRAPH PROPERTIES

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An additive hereditary property of graphs is any class of simple graphs, which is closed under union, subgraphs and isomorphisms. By $\mathbf{L}^a$ we denote a class of all such properties. Let $\mathcal{P}_1, \ldots, \mathcal{P}_n \in \mathbf{L}^a$ and G, $H = (\{v_1, v_2, \ldots, v_n\}, E)$ be graphs. An $H[\mathcal{P}_1, \ldots, \mathcal{P}_n]$-*partition* of G is defined as a $(\mathcal{P}_1, \mathcal{P}_2, \ldots, \mathcal{P}_n)$-partition $(V_1, \ldots, V_n)$ of G satisfying that the existence of $\{x_i, x_j\} \in E(G)$ with $x_i \in V_i$, $x_j \in V_j$, $i \neq j$ implies the existence of $\{v_i, v_j\} \in E$.

A symbol $H[\mathcal{P}_1, \ldots, \mathcal{P}_n]$ denotes a class of all graphs possessing an $H[\mathcal{P}_1, \ldots, \mathcal{P}_n]$-partition. Of course $H[\mathcal{P}_1, \ldots, \mathcal{P}_n] \in \mathbf{L}^a$.

For a given graph H, we say that a property $\mathcal{P} \in \mathbf{L}^a$ is H-reducible over $\mathbf{L}^a$ if $\mathcal{P} = H[\mathcal{P}_1, \ldots, \mathcal{P}_n]$, $n \geq 2$, $\mathcal{P}_1, \ldots, \mathcal{P}_n \in \mathbf{L}^a$ and there exists a graph $G \in \mathcal{P}$ such that for each $H[\mathcal{P}_1, \ldots, \mathcal{P}_n]$-*partition* $(V_1, \ldots, V_n)$ of G we have $V_i \neq \emptyset$ for $i \in [n]$.

We prove that if $\delta(H) \geq 1$ then the number of minimal forbidden subgraphs of H-reducible property $\mathcal{P}$ is infinite, which generalize the result of A. Berger [1]. Moreover, we give many sufficient conditions for infinity of a class of all minimal forbidden subgraphs of $\bar{K}_n$-reducible property. Such properties are joins of non-comparable properties in the lattice $(\mathbf{L}^a, \subseteq)$ and they cover H-reducible properties with $\delta(H) = 0$.

Keywords: generalized colourings of graphs, graph-reducibility, minimal forbidden subgraphs.

AMS Subject Classification: 05C15, 05C75.

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A SELF-STABILIZING ALGORITHM FOR AN ADJACENT VERTEX DISTINGUISHING EDGE-COLOURING OF PLANAR GRAPHS

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Graph edge-colouring is an interesting problem. The objective is to assign colours to edges of the graph such that no two neighbouring edges have the same colour. An adjacent vertex distinguishing edge-colouring or an *AVD*-colouring of a simple graph G is a proper edge-colouring of G such that no pair of adjacent vertices have the same set of colours [1, 2]. K-*AVD*-colouring is an *AVD*-colouring using at most k colours. Let χ_a' be the minimum number of colours in an *AVD*-colouring of G. Finding an optimal colouring, using the least possible number of colours, of an arbitrary graph is a *NP*-complete problem. Of particular interest is the class of planar graph that have received substantial attention so far.

Self-stabilizing colouring problems was investigated in recent years [4, 5]. The concept of self-stabilization, introduced by *Dijkstra* in 1974 [3], is a distributed algorithm that can start from any initial (legitimate or illegitimate) state and guarantees to converge to a legitimate state in a finite time. A self-stabilizing system will eventually correct itself from transient faults automatically without the need for an outside intervention. Once it is in a legitimate state, it stays in it for any subsequent fault free execution.

In this paper, we focus on the *AVD*-colouring problem in a self-stabilizing system of a planar graph. We want to assign each edge a colour such that for any pair of adjacent vertices x and y, the set of colours incident to x is not equal to the set of colours incident to y. The system is said to be in a legitimate state if and only if it has an *AVD*-colouring. Furthermore, we assume that there is a *central daemon model.* It evaluates all the guards, and arbitrarily selects one privileged node with a true guard to complete its corresponding action. We employ node labelling concept. In fact, colours are assigned to edges according to their priorities. The colouring is a $(\Delta + 8)$-*AVD*-colouring of G.

Keywords: graph property, additive, induced-hereditary, vertex partitions, uniquely colourable graphs.

AMS Subject Classification: 05C15, 05C35, O5C75.

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HAMILTONICITY AND TRACEABILITY OF ORIENTED GRAPHS

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A (di)graph is called k-*traceable* if each of its induced sub(di)graphs of order k is traceable. An n-traceable (di)graph of order n is simply called *traceable*. It follows from Dirac's degree condition for hamiltonicity that for $k \geq 2$ every k-traceable graph of order at least $2k-1$ is hamiltonian. We show that the same is true for strong oriented graphs when $k = 2, 3, 4$ but not when $k \geq 5$. We consider the following conjecture.

The Traceability Conjecture: For $k \geq 2$ every k-traceable oriented graph of order at least $2k-1$ is traceable.

We prove that the Traceability Conjecture is true for $k \leq 5$ and that for $k \geq 6$ every strong k-traceable oriented graph of order at least $6k-20$ is traceable.

ON SOME TURÁN AND RAMSEY NUMBERS FOR WHEELS

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Turán number $T(n, G)$ is the maximum number of edges in any n-vertex graph which does not contain a subgraph isomorphic to G. For given graphs $G_1, G_2, \ldots, G_k, k \geq 2$, the *multicolor Ramsey number* $R(G_1, G_2, \ldots, G_k)$ is the smallest integer n such that if we arbitrarily color the edges of complete graph on n vertices with k colors, there is always a monochromatic copy of G_i colored with i, for some $1 \leq i \leq k$. Let W_k be the wheel on k vertices. In the paper we show the exact values and bounds for Turán numbers for wheels. In addition, we give some values and bounds for Ramsey numbers of different graphs versus wheels. In this paper we present new other results in this field as well as some conjectures.

Keywords: edge coloring, Ramsey numbers, Turán numbers.
AMS Subject Classification: 05C15, 05C55.

EQUITABLE COLORING OF KNESER GRAPHS

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The Kneser graph $K(n,k)$ is the graph whose vertices correspond to k-element subsets of set $\{1,2,\ldots,n\}$, and two vertices are adjacent if and only if they represent disjoint subsets.

In 1955, Kneser [1] conjectured that $\chi(K(n,k)) \geq n-2k+2$, $n \geq 2k \geq 2$, where $\chi(G)$ denotes the chromatic number of a given graph G, which was verified by Lovasz in 1978 [2].

Theorem 1 ([2]). *Let $K(n,k)$ be a Kneser graph and $n \geq 2k \geq 2$. Then*

$$\chi(K(n,k)) = n-2k+2. \tag{1}$$

In this paper we study the problem of equitable coloring of Kneser graphs, namely, we establish the equitable chromatic number for $K(n,2)$ and $K(n,3)$. In addition, equitable coloring of some reduced Kneser graphs is considered. A graph G is said to be *equitably k-colorable* if its vertices can be partitioned into k classes $I_1, I_2, \ldots, I_k$ such that each I_i is an independent set and the condition $|\#I_i - \#I_j| \leq 1$ holds for every i,j, where $\#S$ denotes the cardinality of a given set S. Such partition $I_1, I_2, \ldots, I_k$ is called an *equitable partition.* The smallest integer k for which G is equitably k-colorable is known as the *equitable chromatic number* of G and denoted by $\chi_=(G)$.

Our main results are the following theorems.

Theorem 2. *Let $K(n,2)$, $n \geq 4$, be a Kneser graph. Then*

$$\chi_=(K(n,2)) = \begin{cases} n-2 & \text{if } n = 4,5,6, \\ n-1 & \text{otherwise.} \end{cases}$$

Theorem 3. *Let $K_r(n,2)$ be an r-reduced Kneser graph and let $n \geq 2r$. Then*

$$\chi_=(K_r(n,2)) \begin{cases} = n-r & dla\ r = 2, \\ \leq n-r & dla\ r \in \{3,4\}. \end{cases}$$

Theorem 4. *Let $K(n,3)$, $n \geq 6$, be a Kneser graph. Then*

$$\chi_=(K(n,3)) = \begin{cases} n-4 & \text{if } 6 \leq n \leq 13, \\ n-3 & \text{if } n \in \{14,15\}, \\ n-2 & \text{otherwise.} \end{cases}$$

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ON NEARLY THIRD PARTS OF A COMPLETE 2-GRAPH

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By a 2-graph we mean a multigraph with an edge multiplicity at most two. Given an integer t and complete 2-graph ${}^2K_n$ of order n we study packings of t copies of a multigraph F in ${}^2K_n$. We are looking for such an F that the remainder R of the packing, $R \subseteq E({}^2K_n)$, has the smallest possible size $|R|$, $|R| = n(n-1) \mod t$ where $n(n-1)$ is the size of ${}^2K_n$. Each such packing is a decomposition of ${}^2K_n - R$ into parts isomorphic to F. Then F is a tth part of ${}^2K_n - R$, R is called *t-remainder in ${}^2K_n$*, and F is called a *nearly tth part of ${}^2K_n$ with remainder R.*

$$\left\lfloor \frac{{}^2K_n}{t} \right\rfloor_R := \left\{F\colon F \text{ is } t\text{th part of } {}^2K_n \text{with } t\text{-remainder } R\right\}.$$

If $F \in \left\lfloor \frac{{}^2K_n}{t} \right\rfloor_R$ for each remainders R then F is called *nearly tth part of ${}^2K_n$*.

$$\left\lfloor \frac{{}^2K_n}{t} \right\rfloor := \bigcap_R \left\lfloor \frac{{}^2K_n}{t} \right\rfloor_R.$$

The question is if the class $\left\lfloor \frac{{}^2K_n}{t} \right\rfloor$ is nonempty?
We are going to present known results, especially for nearly third parts.

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GRAPHS WITH MAXIMAL NUMBER OF HAMILTONIAN k-SETS

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A *hamiltonian decomposition* of G is a collection of edge-disjoint hamiltonian cycles whose union is equal to G. The hamiltonian decomposition into k parts is called a *hamiltonian k-set*, $k \geq 2$.

Let $h_k(G)$ be the *number* of hamiltonian k-sets of G and let $h_k(n)$ denote the maximum of $h_k(G)$ if G ranges over all multigraphs of order n. Then $h_k(G) = 0$ if G is not $2k$-valent. We are going to study the largest values of the function h_k. The related problem for $k = 2$ of determining the minimal positive number of hamiltonian pairs has been studied by several authors, see [1, 3, 4]. In particular, it follows from Thomason's [4] that $h_2(G) \geq 4$ if G has a hamiltonian pair. This lower bound is sharp because multigraphs with exactly four hamiltonian pairs have been found by Skupień [2, 3] for each $n \geq 3$.

Our aim is to contribute to the open problem stated in [2] on the corresponding upper bounds. For $n \geq 4$, we characterize n-vertex multigraphs with two largest values of h_k, namely with $h_k \in \{k!^{n-1}, k!^{n-1}/k\}$.

Keywords: tree, multigraph, hamiltonian decompositions.

AMS Subject Classification: 05C70, 05C35, 05C38, 05C45.

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ON NORMAL ODD PARTITIONS IN CUBIC GRAPHS

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A *normal partition* of the edges of a cubic graph is a partition into *trails* (no repeated edge) such that each vertex is the end vertex of exactly one trail of the partition. We give here some results and propose a conjecture related to perfect matchings in cubic graphs.

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ON ODD AND SEMI-ODD LINEAR PARTITIONS OF CUBIC GRAPHS

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A *linear forest* is a graph whose connected components are chordless paths. A *linear partition* of a graph G is a partition of its edge set into linear forests and its linear arboricity, $la(G)$, introduced by Harary [3], is the minimum number of linear forests in a linear partition.

In this paper we consider linear partitions of cubic simple graphs for which it is well known [1] that $la(G) = 2$. A linear partition $L = (L_B, L_R)$ is said to be *odd* whenever each path of $L_B \cup L_R$ has odd length and *semi-odd* whenever each path of L_B (or each path of L_R) has odd length.

In [2] Aldred and Wormald showed that a cubic graph G is 3-edge colourable if and only if G has an odd linear partition. We give here more precise results and we give as an application some partial results about a conjecture of Fouquet (1991) on *compatible linear partition*. Moreover, we prove that a cubic graph has a semi-odd linear partition if and only if it has a perfect matching.

Keywords: cubic graph, edge-colouring, linear partition, matching.

AMS Subject Classification: 05C38, 05C15, 05C70.

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DIRECTED HYPERGRAPHS: A TOOL FOR RESAEARCHING DIGRAPHS AND HYPERGRAPHS

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In this work we introduce the concept of *directed hypergraph*. It is a generalisation of the concept of digraph and is closely related with hypergraphs. The basic idea is to take a hypergraph, partition its hyperedges non-trivially (when possible), and give a total order to such partitions. The elements of these partitions are called *levels*. In order to preserve the structure of the underlying hypergraph, we ask that only vertices which belong to exactly the same hyperedges may be in the same level of any hyperedge they belong to. Some little adjustments are needed to avoid directed walks within a single hyperedge of the underlying hypergraph, and to deal with isolated vertices.

The concepts of independent set, absorbent set, and transversal set are inherited directly from digraphs. Up to now, our efforts have been directed to the study of transversal kernels (that is, sets which are independent, absorbent, and transversal) in directed hypergraphs. However, we think that the concept of a directed hypergraph may be useful for studying other aspects of digraphs and hypergraphs.

As a consequence of our results on this topic, we have found both a class of kernel-perfect digraphs with odd cycles and a class of hypergraphs which have a strongly independent transversal set.

KERNELS BY MONOCHROMATIC DIRECTED PATHS IN 3-QUASI-TRANSITIVE DIGRAPHS

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For general concepts we refer the reader to [1]. Let D be a digraph, $V(D)$ and $A(D)$ will denote the set of vertices and arcs of D, repectively. We call the digraph D an m-coloured digraph if the arcs of D are coloured with m colours. A directed path is called monochromatic if all of its arcs are coloured alike.

A set $I \subseteq V(D)$ is independent if $A(D[I]) = \phi$. A kernel N of D is an independent set of vertices such that for each $z \in V(D)-N$ there is a zN-arc in D.

A set $N \subseteq V(D)$ is said to be a kernel by monochromatic paths if for every pair of different vertices u, $v \in N$, there is no monochromatic directed path between them and for every vertex $x \in V(D) - N$, there is a vertex $y \in N$ such that there is an xy-monochromatic directed path. The concept of kernel by monochromatic paths is a generalization to the one of kernel. The problem of the existence of a kernel by monochromatic paths in a given m-coloured digraph has been studied by several authors for example Galeana-Sánchez [2, 3], Galeana-Sánchez and García-Ruvalcaba [4, 5], Galeana-Sánchez and Pastrana [7], Galeana-Sánchez and Rojas-Monroy [8, 9, 10], S. Minggang [11] and Sands, Sauer and Woodrow [12]. Almost of these results are concerning to tournaments or digraphs near to tournaments.

A digraph is transitive whenever $(u, v) \in A(D)$ and $(v, w) \in A(D)$ implies $(u, w) \in A(D)$. A digraph D is called quasi-transitive when $(u, v) \in A(D)$ and $(v, w) \in A(D)$ implies $(u, w) \in A(D)$ or $(w, u) \in A(D)$. This concept was introduced by Ghouilá-Houri in 1962 [14] and has been studied by several authors in particular Bang-Jensen and Huang [1, 2, 3], Huang [15], Skrien [19]. It was proved by Ghouilá-Houri [14] that an undirected graph can be oriented as a quasi-transitive digraph if and only if it can be oriented as a transitive digraph, namely comparability graph. More information about comparability graphs can be found in [13, 16].

In this work is introduced the concept of a 3-quasi-transitive digraph and is given some sufficient conditions for the existence of kernels by monochromatic paths in such m-coloured digraphs.

Keywords: kernel, kernel by monochromatic paths, quasi-transitive digraphs, 3-quasitransitive digraphs.
AMS Subject Classification: 05C20.

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SEMIKERNELS AND KERNELS BY MONOCHROMATIC DIRECTED PATHS IN EDGE-COLOURED BIPARTITE TOURNAMENTS

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For general concepts we refer the reader to [1]. Let D be a digraph, $V(D)$ and $A(D)$ will denote the set of vertices and arcs of D, repectively. We call the digraph D an m-coloured digraph if the arcs of D are coloured with m colours. A directed path is called monochromatic if all of its arcs are coloured alike.

A set $I \subseteq V(D)$ is independent if $A(D[I]) = \phi$. A kernel N of D is an independent set of vertices such that for each $z \in V(D) - N$ there is a zN-arc in D. An important concept in the development of Kernel Theory has been the one of semikernel that was introduced by V. Neumann-Lara [6]. A set $S \subseteq V(D)$ is a semikernel of D if it is an independent set and if $(s, u) \in A(D)$ with $s \in S$ and $u \in V(D) - S$ then there is an arc from u to some vertex in S.

A set $N \subseteq V(D)$ is said to be a kernel by monochromatic paths if for every pair of different vertices u, $v \in N$, there is no monochromatic directed path between them and for every vertex $x \in V(D) - N$, there is a vertex $y \in N$ such that there is an xy-monochromatic directed path. The concept of kernel by monochromatic paths is a generalization to the one of kernel. The problem of the existence of a kernel by monochromatic paths in a given m-coloured digraph has been studied by several authors for example Galeana-Sánchez [2, 3], Galeana-Sánchez and García-Ruvalcaba [4, 5], Galeana-Sánchez and Pastrana [7], Galeana-Sánchez and Rojas-Monroy [8, 9, 10], S. Minggang [11] and Sands, Sauer and Woodrow [12]. Almost of these results are concerning to tournaments or digraphs near to tournaments.

A digraph D is called a bipartite tournament if its vertices can be partitioned into two sets V_1 and V_2 such that every arc of D has an endpoint in V_1 and the other endpoint in V_2 and for every $x_1 \in V_1$ and every $x_2 \in V_2$, we have $|\{(x_1, x_2), (x_2, x_1)\} \cap A(D)| = 1$. If $u \in V(D)$ we denote by $A^+(u)$ the set of arcs $\{(u, v)/v \in V(D)\}$ and we say that $A^+(u)$ is monochromatic if all of its elements have the same colour. T_4 is the bipartite tournament defined as follows: $V(T_4) = \{u, v, w, x\}$, $A(T_4) = \{(u, v), (v, w), (w, x), (u, x)\}$. We say that a 3-coloured digraph H is a $(1, 1, 2)$ subdivision of C_3 (the directed cycle of length 3) if H is a directed cycle of length 4, $(u_1, u_2, u_3, u_4, u_1)$, such that (u_1, u_2) is coloured a, (u_2, u_3) is coloured b and the arcs (u_3, u_4) and (u_4, u_1) are coloured c, with $a \neq b$, $b \neq c$ and $a \neq c$.

In this work we defined the concept of semikernel module i (i is a colour) that is related to the one of semikernel. We used this concept to proved that if T is a bipartite tournament such that $A^+(u)$ is monochromatic for every vertex u, every subdigraph of T isomorphic to T_4 is at most 2-coloured and T has no $(1, 1, 2)$ subdivisions of C_3 then T has a kernel by monochromatic paths.

Keywords: kernel, kernel by monochromatic paths, bipartite tournaments.

AMS Subject Classification: 05C20.

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LIST-COST COLORING OF VERTICES AND/OR EDGES OF SOME SPARSE GRAPHS

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We consider a list-cost coloring of vertices and edges in the model of vertex, edge, total and pseudototal coloring of graphs. We use a dynamic programming approach to derive polynomial-time algorithms for solving the above problems for trees. Then we generalize this approach to arbitrary graphs with bounded cyclomatic numbers, to multitrees and to linear hypertrees.

A NOTE ON AN EMBEDDING PROBLEM IN TRANSITIVE TOURNAMENTS

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Let TT_n be a transitive tournament on n vertices. It is known [1] that for any directed acyclic graph $\overrightarrow{G}$ of order n and of size not greater than $\frac{3}{4}(n-1)$ two directed graphs isomorphic to $\overrightarrow{G}$ are arc disjoint subgraphs of TT_n. We consider a problem of embedding of a graph into its complement in a transitive tournament. We show that any directed acyclic graph $\overrightarrow{G}$ of size not greater than $\frac{2}{3}(n-1)$ is embeddable into its complement in TT_n. Moreover, this bound is generally the best possible.

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COLOURING (a, b)-DISTANCE GRAPHS

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Exoo [1] introduced a variation on the famous Nelson-Hadwiger problem on colouring the Euclidean plane: what is the minimum number of colours needed to colour the plane such that points in euclidian distance between $1 - \varepsilon$ and $1 + \varepsilon$ get different colours. He gives some bounds for some values of ε and offers a conjecture that the number is at least 7 for $\varepsilon > 0$. We prove that that number is at least 5 and give some new bounds for different values of ε.

Keywords: colouring, Nelson-Hadwiger problem.

AMS Subject Classification: 05C15.

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ON GRAPHS WITH EQUAL 2-DOMINATION AND DOMINATION NUMBERS

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Let G be a simple graph, and let p be a positive integer. A subset $D \subseteq V(G)$ is a *p-dominating set* of the graph G, if every vertex $v \in V(G) - D$ is adjacent to at least p vertices in D. The ***p-domination number*** $\gamma_p(G)$ is the minimum cardinality among the p-dominating sets of G. Note that the 1-domination number $\gamma_1(G)$ is the usual ***domination number*** $\gamma(G)$.

In 1985, Fink and Jacobson showed that, if $p \geq 2$ is an integer and G is a graph with $p \leq \Delta(G)$, then $\gamma_p(G) \geq \gamma(G) + p - 2$. This theorem implies that $\gamma_p(G) > \gamma(G)$ when $p \geq 3$. However, in the case $p = 2$ the equality $\gamma_2(G) = \gamma(G)$ is possible. We will present some sufficient as well as some necessary conditions for graphs G with the property that $\gamma_2(G) = \gamma(G)$ and we will analyze some graph classes with respect to this equality.

Keywords: domination, 2-domination.

AMS Subject Classification: 05C69.

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COLORABILITY OF MIXED HYPERGRAPHS AND THEIR CHROMATIC INVERSES

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A mixed hypergraph is a triple $H = (X, C, D)$, where X is the set of vertices, and C and D are the families of 'C-edges' and 'D-edges', respectively. Each C-edge and D-edge is a subset of X with at least two elements. The coloring of H is proper if

- each C-edge has at least two vertices with a common color, and
- each D-edge has at least two vertices with different colors.

A mixed hypergraph is called colorable if it admits at least one proper coloring; and otherwise it is termed uncolorable.

The chromatic inverse of H is the mixed hypergraph $H^c = (X, D, C)$, i.e., where each C-edge becomes a D-edge and vice versa. In this talk we prove that if $P \neq NP$, there exists no good characterization or polynomial-time algorithm to recognize those mixed hypergraphs H such that both H and its chromatic inverse H^c are colorable, or both are uncolorable. This assertion is valid for the restricted class of 3-uniform mixed hypergraphs, too. Our theorem answers a problem raised implicitly by Voloshin [Australas. J. Combin. 1995] and explicitly by Voloshin and the second author [Discrete Applied Math. 2000].

Keywords: mixed hypergraph, chromatic inverse, colorable, uncolorable.

AMS Subject Classification: 05C15.

ON THE GLOBAL AND LOCAL STRUCTURE OF 1-PLANAR GRAPHS

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A graph is said to be 1-planar if there exists its drawing in the plane such that each edge is crossed by at most one another edge. We study various aspect of global and local structure of 1-planar graphs; we complete results on the maximum number of edges of small 1-planar graphs and explore some extremal families (according to minimum vertex degree and girth) of 1-planar graphs from the point of view of existence small light graphs.

Keywords: 1-planar graph, light graph.
AMS Subject Classification: 05C10.

RESULTS ON γ_t-CRITICAL GRAPHS

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A graph G with no isolated vertex is total domination vertex critical if for any vertex v of G that is not adjacent to a vertex of degree one, the total domination number of $G - v$ is less than the total domination number of G. These graphs we call γ_t-critical. If such a graph G has total domination number k, we call it k-γ_t-critical. In this paper we introduce a class of k-γ_t-critical graphs for any positive integer $k \geq 3$ and study some problems of Goddard, Haynes, Henning, and van der Merwe concerning γ_t-critical graphs.

Keywords: total domination, vertex critical, diameter.
AMS Subject Classification: 05C69.

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RAINBOW FACES IN EDGE COLORED PLANE GRAPHS

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A face of an edge colored plane graph is called *rainbow* if all its edges receive distinct colors. The maximum number of colors used in an edge coloring of a connected plane graph G with no rainbow face is called *the edge-rainbowness* of G.

We prove that the edge-rainbowness of G equals to the maximum number of edges of a connected bridge face factor H of G, where a *bridge face factor* H of a plane graph G is a spanning subgraph H of G in which every face is incident with a bridge and the interior of any one face $f \in F(G)$ is a subset of the interior of some face $f' \in F(H)$. We also show upper and lower bounds on the edge-rainbowness of graphs based on edge connectivity, girth of the dual G^* and other basic graph invariants. Moreover, we present infinite classes of graphs where these equalities are attained.

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LIGHT GRAPHS — A SURVEY

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It is well known that every planar graph contains a vertex of degree at most 5. A beautiful theorem of Kotzig [3] states that every 3-connected planar graph contains an edge whose endvertices have degree-sum at most 13. Fabrici and Jendrol' [2] proved that every 3-connected planar graph G that contains a k-path, a path on k vertices, contains also a k-path P such that every vertex of P has degree at most $5k$. A beautiful result by Enomoto and Ota [1] says that every 3-connected planar graph G of order at least k contains a connected subgraph H of order k such that the degree sum of vertices of H in G is at most $8k-1$. Motivated by these results, a concept of light graphs has been introduced. A graph H is said to be *light* in a class $\mathcal{G}$ of graphs if at least one member of $\mathcal{G}$ contains a copy of H and there is an integer $w(H,\mathcal{G})$ such that each member G of $\mathcal{G}$ with a copy of H also has a copy of H with degree sum $\sum_{v\in V(H)} deg_G(v) \leq w(H,\mathcal{G})$.

We will present a survey of results on light graphs in different families of planar graphs.

Keywords: planar graphs, polyhedral maps, light subgraphs, path, cycle, embeddings, subgraphs with restricted degrees, triangulation.

AMS Subject Classification: 05C10, 52B10, 05C38, 57N05.

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FINDING COVERING AND PARTITION IN TIME $O(2^n)$

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Björklund and Husfeldt introduced very simple and fast algorithm based on inclusion-exclusion principle answering the question is there a covering of given set by k sets from given family of its subsets. It has many applications e.g.: to graph colouring. In the general case the algorithm only says if there is covering, but does not say how to find it. However in case of graph colouring Björklund and Husfeldt gave an algorithm that finds a proper colouring by finding chromatic numbers of a sequence of graphs built on the given one. Our main result is an algorithm based on the same methods finding a covering (partition) in general case.

Keywords: covering, partition, colouring.

AMS Subject Classification: primary 05B40, secondary 05C15.

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ORE-TYPE CONDITIONS FOR ON-LINE ARBITRARILY VERTEX DECOMPOSABLE GRAPHS

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A graph $G = (V, E)$ of order n is called *arbitrarily vertex decomposable* if for every sequence $(n_1, \ldots, n_k)$ of positive integers such that $n_1 + \cdots + n_k = n$ there exists a partition $(V_1, \ldots, V_k)$ of the vertex set V such that V_i induces a connected subgraph of order n_i for each $i = 1, \ldots, k$.

Clearly, each path, and hence each treaceable graph, is arbitrarily vertex decomposable. Recently, Horňák, Marczyk, Schiermeyer, and Woźniak proved the following Ore-type result.

Theorem 1. *Let G be a two-connected graph of order $n \geq 11$ that admits a perfect matching or a matching omitting exactly one vertex. If the degree sum of every pair of nonadjacent vertices is at least $n-4$, then G is arbitrarily vertex decomposable.*

In my talk, the on-line version of the problem will be considered.

TOTAL COLORINGS OF CARTESIAN PRODUCTS OF GRAPHS

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The total chromatic number $\chi''(G)$ of a graph G is defined to be the minimum number of colors in an assignment of colors to the elements (vertices and edges) of G such that neighbored elements (two adjacent vertices or two adjacent edges or a vertex and an incident edge) must be colored differently. We investigate the total chromatic number of cartesian products $K_n \times K_m$ of complete graphs, $C_n \times C_m$ of cycles and $K_n \times H$ as well as $C_n \times H$ where H is a bipartite graph.

Keywords: cartesian graph product, total chromatic number.

AMS Subject Classification: 05C15.

UNIVERSAL SIXTH PARTS OF A COMPLETE GRAPH

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We consider an edge-decomposition of a complete n-vertex graph K_n into t isomorphic parts so that a possible edge-remainder R is as small as possible. Namely, $|R| = \binom{n}{2} \bmod t$. The general conjecture [3] says that there exists a graph F (called a universal tth part of K_n) such that for each R the graph $K_n - R$ is edge-decomposable into t copies of F. The conjecture has been proved in some cases, e.g. if $|R| = 1$ [2] and $t \leq 5$ [4]. Our aim is to show that the conjecture is true for $t = 6$.

Keywords: minimum remainder, decomposition part, decomposition matrix.
AMS Subject Classification: 05C70, 05C50.

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LATTICE AND ISOMETRIC DIMENSION OF PARTIAL CUBES

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A graph G is called a partial cube if it is an isometric subgraph of some hypercube Q_n. Equivalently, G is a partial cube if it can be isometrically embedded into the integer lattice $\mathbb{Z}^d$. The smallest such n and d are called the isometric dimension and the lattice dimension of G, respectively.

The lattice dimension for some special families of partial cubes has been determined in [1] and [3]. In [2] the so-called semicube graph of a partial cube has been introduced and a polynomial time algorithm for finding the lattice dimension of a partial cube has been described.

We characterize which graphs are semicube graphs of partial cubes. Using results from [4] partial cubes with equal isometric and lattice dimension are characterized.

Keywords: partial cube, isometric dimension, lattice dimension.

AMS Subject Classification: 05C75, 05C12, 05C62.

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ON THE CROSSING NUMBERS OF PRODUCTS OF STARS

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The *crossing number* $cr(G)$ of a graph G is the minimum possible number of edge crossings in a drawing of G in the plane. Computing the crossing number of a given graph is in general an elusive problem. Garey and Johnson have proved that this problem is NP-complete. The exact value of the crossing number is known only for some families of graphs. The structure of Cartesian products of graphs makes Cartesian products of special graphs one of few graph classes, for which exact crossing number results are known.

In 1973, Harary, Kainen, and Schwenk established the crossing number of $C_3 \times C_3$ and conjectured that $cr(C_m \times C_n) = m(n-2)$ for $3 \leq m \leq n$. Recently has been proved by Glebsky and Salazar that for any fixed $m \geq 3$, the conjecture holds for all $n \geq m(m+1)$. Besides the Cartesian products of two cycles, there are several other exact results. In 2006, Bokal proved the conjecture given by Jendrol' and Ščerbová that $cr(K_{1,n} \times P_m) = (m-1) \lfloor \frac{n}{2} \rfloor \lfloor \frac{n-1}{2} \rfloor$ for the path P_m of length m. Applying the capped Cartesian product operation in combination with a newly introduced π-subdivision he established the crossing numbers for the Cartesian products of $K_{1,n}$ with a tree of maximum degree 3 and for the product $W_n \times P_m$, where W_n is the wheel on $n+1$ vertices.

In the talk, we summarise the known crossing numbers for the Cartesian products of stars with small graphs. We present the latest results on crossing numbers of multipartite complete graphs and we give new values of crossing numbers of products of stars with other graphs.

Keywords: graph, star, drawing, crossing number.

AMS Subject Classification: 05C10.

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ON PAGE NUMBER OF N-FREE POSETS

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The book embedding of graphs was introduced by Bernhart and Kainen in 1979. In some applications, the book embedding problem for arbitrary graphs reduces to that for covering graphs of partially ordered sets (posets) over the set of all linear extensions. We study here the book embedding of posets, in particular, for some N-free posets.

Let P denote a poset and $G(P)$ its covering graph. A complete characterization of posets with page number 1 is known — $p(P) = 1$ if and only if $G(P)$ has no cycle, i.e., $G(P)$ is a forest. A complete characterization of posets with page number 2 is an open question, it is evident only that such posets must be planar.

Nowakowski and Parker (1989) have provided some bounds for the page number in general and also for planar posets, and Sysło (1990) has proved some other bounds, in particular investigating the relation between the page number and the jump number of a poset. Only some partial results for planar posets are known and it is still an open question whether the page number is bounded for such posets.

An N is a poset consisting of four elements a, b, c, d such that $a < c$, $b < c$, $b < d$ are the only comparabilities. This poset plays an important role in studying several algorithmic problems on posets. A poset P is *N-free* if its covering digraph (i.e., its Hasse diagram) contains no subdigrah isomorphic to the covering digraph of N. N-free posets have been introduced by Grillet (1969) as posets having the *C.A.C. property*, i.e., each maximal chain meets each maximal antichain.

An N-free poset P can be defined also as a line digraph. Making use of characterizations of line digraphs we provide some partial results for the page number problem on some N-free posets: with tree-like root digraphs (exact value), arbitrary N-free (bounds), planar N-free.

SMALL TRANSVERSALS IN HYPERGRAPHS

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A transversal in a hypergraph is a set of vertices that meets all the edges. We discuss the problem of determining the smallest cardinality of a transversal for some kinds of hypergraphs. In particular, we present our recent results concerning a variant of greedy algorithm which constructs small transversals. Finally, we show a relationship between these results and some other combinatorial problems.

DISTANCE DOMINATION AND DISTANCE IRREDUNDANCE IN GRAPHS

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A set $D \subseteq V$ of vertices is said to be a *(connected) distance k-dominating set* of G if the distance between each vertex $u \in V - D$ and D is at most k (and D induces a connected graph in G). The minimum cardinality of a (connected) distance k-dominating set in G is the *(connected) distance k-domination number* of G. The set D is defined to be a *total k-dominating set* of G if every vertex in V is within distance k from some vertex of D other than itself. The minimum cardinality among all total k-dominating sets of G is called the *total k-domination number*. For $x \in X \subseteq V$, if $N^k[x] - N^k[X - x] \neq \emptyset$, the vertex x is said to be *k-irredundant in X*. A set X containing only k-irredundant vertices is called *k-irredundant*. The *k-irredundance number of G* is the minimum cardinality taken over all maximal k-irredundant sets of vertices of G. We establish lower bounds for the distance k-irredundance number and some distance domination parameters, especially the distance k-domination number and total k-domination number, of graphs and trees. In addition, we present classes of examples that show that these bounds are sharp. Our results generalize a result of Meierling and Volkmann [6] and Cyman, Lemańska and Raczek [1] and results of Favaron and Kratsch [3].

Keywords: domination, irredundance, distance domination number, total domination number, connected domination number, distance irredundance number, tree.

AMS Subject Classification: 05C69.

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k-CYCLE FREE ONE-FACTORIZATIONS OF COMPLETE GRAPHS

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A one-factorization of a regular graph G is *uniform* if the union of any two one-factors is isomorphic to the same two-factor H, which is a disjoint union of even cycles. There are only several infinite classes of known uniform one-factorizations of complete graphs.

An opposite problem may be dealt with; one can ask about the existence of one-factorization such that the union of any two one-factors does not include cycles of given lengths. A one-factorization $F = \{F_1, F_2, \ldots, F_t\}$ of G is said *k-cycle free* if the union of any two one-factors does not include the cycle C_k as a component. Consequently, F is *k^*-cycle free* if the union of any two one-factors does not include all cycles of lengths $\leq k$.

It is proved that for every $n \geq 3$ and every even $k \geq 4$, where $k \neq 2n$, there exists a k-cycle free one-factorization of the complete graph K_{2n}. Moreover, some infinite classes of k^*-cycle free one-factorizations of K_{2n} are constructed.

GENERALIZED COLOURING AND THE EXISTENCE OF UNIQUELY COLORABLE GRAPHS

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Let $\mathcal{P}_1, \mathcal{P}_2, \ldots, \mathcal{P}_n$ be a graph properties. A vertex $(\mathcal{P}_1, \mathcal{P}_2, \ldots, \mathcal{P}_n)$-colouring of a graph G is a partition $\{V_1, V_2, \ldots, V_n\}$ of its vertex set $V(G)$ into n classes such that each V_i induces a subgraph $G[V_i]$ with property $\mathcal{P}_i$. A graph G is said to be uniquely $(\mathcal{P}_1, \mathcal{P}_2, \ldots, \mathcal{P}_n)$-colourable, $n \geq 2$, if G has exactly one $(\mathcal{P}_1, \mathcal{P}_2, \ldots, \mathcal{P}_n)$-colouring. We will present a short survey on the existence of uniquely colourable graphs with respect to additive induced-hereditary properties. Analogous questions for countable graphs will be raised. It will be shown that each reducible graph property has a generator, which is uniquely $(\mathcal{P}_1, \mathcal{P}_2, \ldots, \mathcal{P}_n)$-colourable.

Keywords: graph property, additive, induced-hereditary, vertex partitions, uniquely colourable graphs.

AMS Subject Classification: 05C15, 05C35, O5C75.

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STRONG GENERATORS OF HOM-PROPERTIES

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For a simple graph H, $\rightarrow H$ denotes the class of all graphs that admit homomorphisms to H (such classes of graphs are called *hom-properties*). A *strong generator* of the class $\mathcal{P}$ of graphs is a graph G such that for every graph G^* belonging to the class $\mathcal{P}$ there exists a proper subgraph of G isomorphic to G^* (see e.g. [4]).

We investigate hom-properties from the point of view of the lattice of hereditary properties of graphs (see also [1, 2, 3]). In particular, we are interested in a characterization of the strong generators of $\rightarrow H$. We prove the existence of a strong generator of $\rightarrow H$, for any finite graph H. Moreover, we show that the structure of the strong generator of $\rightarrow H$ preserves the structure of the unique factorization of the hom-property $\rightarrow H = (\rightarrow H_1) \circ (\rightarrow H_2) \circ \cdots \circ (\rightarrow H_n)$, where $H_1, H_2, \ldots H_n$ are indecomposable graphs satisfying $H = H_1 + H_2 + \cdots + H_n$ (see [2]).

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k-INTERSECTION EDGE COLOURING

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We introduce the notion of *k-intersection edge colouring* of a graph G as a proper edge colouring in which any two adjacent vertices do not have more than k pairs of edges where each pair receive the same colour. The minimum number of colours required for such a colouring is called *k-intersection chromatic index* and is denoted by $\chi'_k(G)$. We show that $\chi'_k(G) = O(\Delta^2/k)$ for any G. We also show that there are graphs which require at least $\frac{\Delta^2}{2k}$ colours. Our results hold for each k, $1 \leq k \leq \Delta$. Here, Δ is the maximum degree of G.

In a proper edge colouring, the number of common colours between a pair of adjacent vertices can be as high as $\Delta(G)$. If a proper edge colouring is also a distance-2 edge colouring (which also is an acyclic edge colouring), then the number of common colours is exactly 1. We need $O(\Delta^2)$ colours for a distance-2 edge colouring, while $\Delta + 1$ is an upper bound for a proper edge colouring. k-intersection edge colouring simultaneously generalises both the notions by allowing the maximum number of colours to be bounded by some k between 1 and Δ, inclusive of both. Our study is motivated by the interest to know what happens to the chromatic index when the maximum number of common colours is bounded by k.

Our proofs are based on probabilistic arguments. We show that our bounds are tight up to a constant factor for complete graphs.

It would be interesting to know, if the lower bound is tight for other classes of graphs like bicliques (complete bipartite graphs). Some algorithmic aspects are also being looked at.

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PACKING OF NONUNIFORM HYPERGRAPHS

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Hypergraphs $H_1, \ldots, H_N$ of order n are mutually packable if one can find their edge disjoint copies in the complete hypergraph of the same order. We prove that an arbitrary set of hypergraphs is mutually packable if the product or the sum of their sizes satisfy some upper bound.

Keywords: nonuniform hypergraph, uniform hypergraph, packing.
AMS Subject Classification: 05C65, 05C70, 05D05.

NEIGHBOUR-DISTINGUISHING GRAPH-WEIGHTINGS

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The following modification of a conjecture by M. Karoński, T. Łuczak and A. Thomason will be discussed. Let us assign positive integers to the edges and vertices of a simple graph G. As a result we obtain a vertex-colouring of G by sums of weights assigned to the vertex and its adjacent edges. Can we obtain a proper coloring using only weights 1 and 2 for an arbitrary G?

A positive answer when G is a 3-colourable, complete or 4-regular graph will be provided. We will also present some upper bounds on this parameter. In particular, it is enough to use weights from 1 to 11, as well as from 1 to $\lfloor \frac{\chi(G)}{2} \rfloor + 1$, for an arbitrary graph G. The case of regular graphs will be discussed separately, as well as a more general version of the problem for hypergraphs.

RECENT RESULTS ON DOMINATION AND RELATED PARAMETERS IN REGULAR GRAPHS OF LARGE GIRTH

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In this talk we will survey several recent developments concerning bounds on the domination number and related graph parameters in regular graphs of large girth. The methods used to obtain the different results comprise constructive arguments as well as the asymptotic analysis of random processes on graphs.

We demonstrate how the combination of these different approaches allows for instance to prove that the domination number γ of cubic graphs of order n and girth g satisfies $\gamma \leq 0.3064n + O(n/g)$ which improves several recent results considerably.

WELL PRIMITIVE GRAPHS

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A digraph D is primitive if there exists an integer $k > 0$ such that for all pairs of vertices $u, v \in V(D)$ (not necessarily distinct), there is a directed walk from u to v of length k in D. The least such k is called the exponent of the digraph D, denoted by $\exp(D)$.

Our investigation are restricted to the class of symmetric digraphs. Loops are permitted. Every symmetric digraph can be obtained from a graph by replacing each edge by two arcs, one in each direction. In this sense we consider the primitivity of graphs.

In 1990 Brualdi and Liu [2] introduced generalized exponents of the primitive digraph. One of them is the exponent of the vertex in the primitive digraph.

Let G be a primitive graph and $V(G) = \{v_1, \ldots, v_n\}$. The vertices of G can be relabeled in such a way that $\exp_G(v_1) \leq \exp_G(v_2) \leq \cdots \leq \exp_G(v_n) = \exp(G)$. Well primitive graph is a primitive graph in which $\exp_G(v) = \exp(G)$, for all $v \in V(G)$. We consider some properties of such graphs.

Keywords: primitive symmetric digraph, exponent of primitivity, generalized exponent.

AMS Subject Classification: 05C35, 15A33.

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CLOSURES, CYCLES AND PATHS

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In 1960 Ore proved the following theorem: Let G be a graph of order n. If $d(u) + d(v) \geq n$ for every pair of nonadjacent vertices u and v, then G is hamiltonian. Since then for several other graph properties similar sufficient degree conditions have been obtained, so called "Ore-type degree conditions". In 2000 Faudree, Saito, Schelp and Schiermeyer strengthened Ore's theorem as follows: They determined the maximum number of pairs of nonadjacent vertices that can have degree sum less than n (i.e., violate Ore's condition) but still imply that the graph is hamiltonian. In this talk we will show that for some other graph properties the corresponding Ore-type degree conditions can be strengthened as well. These graph properties include traceable graphs, hamiltonian connected graphs, k-leaf connected graphs, pancyclic graphs and graphs having a 2-factor with two components. Graph closures are computed to show these results.

Keywords: degree condition, closure, cycles, paths.

AMS Subject Classification: 05C38, 05C85.

INDEPENDENT DOMINATION NUMBER OF CARTESIAN PRODUCT OF DIRECTED PATHS AND DIRECTED CYCLES

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The independent domination number of a digraph D, denoted $i(D)$, is the cardinality of the smallest independent dominating set of D.

In this paper we calculate the independent domination number of the *cartesian product* of two *directed Paths* P_m and P_n for arbitrary m and n. Also, we calculate the independent domination number of the *cartesian product* of two *directed cycles* C_m and C_n for $m, n \equiv 0(mod\ 3)$, $n \equiv 0(mod\ m)$ and $m, n \equiv 0(mod\ 2)$. There are many values of m and n such that $C_m \times C_n$ does not have an independent dominating set.

Keywords: directed path, directed cycle, cartesian product, independent domination number.

COLORINGS OF ORIENTED GRAPHS

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An oriented graph is a digraph with no loops and no opposite arcs. An oriented (vertex) k-coloring of an oriented graph G is a partition of the vertex set of G into k subsets in such a way that all the arcs between any two subsets have the same direction. Such a coloring may equivalently be viewed as a homomorphism from G to some oriented graph H on k vertices.

Oriented colorings have been considered by several people for more than a decade. In this talk, we shall survey main results on this topic, including the study of the so-called oriented chromatic number as well as the oriented chromatic index (an oriented arc-coloring of an oriented graph is simply an oriented vertex-coloring of its line (di)graph).

ON EXTREMAL k-INDEPENDENT SETS IN GRAPHS

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Let $k \geq 2$ be an integer. A subset $S \subset V(G)$ is a k-independent set of G if no two of its vertices are in distance less than k. A graph G is called k-well covered if every maximal k-independent set of vertices in G is a maximum k-independent set. We study maximal k-independent sets and maximum k-independent sets in graphs and their products, among others: in the G-join and in the duplication of a subset of vertices of a graph. We also consider the concept of k-well covered graphs which generalizes the concept of well coveredness.

Keywords: independent set, well covered graph, k-well covered graph, graph products.

AMS Subject Classification: 05C15.

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GEODETIC SETS IN MEDIAN AND CARTESIAN PRODUCT GRAPHS

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A set S of vertices of a graph G is a geodetic set if every vertex of G lies in at least one interval between the vertices of S. The size of a minimum geodetic set in G is the geodetic number of G.

In this talk we present results on minimum geodetic sets in median graphs, studied with respect to the operation of peripheral expansion. We show that geodetic number of a median graph G, obtained by the peripheral expansion from a graph H along a convex subgraph P, lies between $g(H)$ and $g(H)+g(P)$ with both bounds being sharp. Along the way geodetic sets of median prisms are considered and median graphs that possess a geodetic set of size two are characterized. Upper bounds for the geodetic number of Cartesian product graphs are proved and for several classes exact values are obtained. It is proved that many metrically defined sets in Cartesian products have product structure and that the contour set of a Cartesian product is geodetic if and only if their projections are geodetic sets in factors. The presented results were obtained jointly with Boštjan Brešar and Sandi Klavžar.

Keywords: median graph, Cartesian product, geodetic number, geodetic set, expansion, contour set.

AMS Subject Classification: 05C12, 05C75.

THREE TOPICS IN EDGE-COLORING: CIRCULAR, INTERVAL, AND PARITY EDGE-COLORINGS

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This talk will present recent results concerning three types of edge-colorings of graphs.

I: *Circular edge-colorings of cartesian products of graphs* (joint work with Xuding Zhu, National Sun Yat-Sen University, Taiwan).

The *circular chromatic index* of a graph G, written $\chi'_c(G)$, is the minimum r permitting a function $f\colon\ E(G) \to [0,r)$ such that $1 \leq |f(e) - f(e')| \leq r - 1$ when e and e' are incident; always $\Delta(G) < \chi'_c(G) \leq \chi'(G)$, where $\chi'(G)$ is the ordinary edge-chromatic number.

Let H be an $(s-2)$-regular graph of odd order; thus $s < \chi'_c(H\Box C_{2m+1}) \leq s+1$, where $\Box$ denotes Cartesian product. For $s \equiv 0 \bmod 4$, we prove that $\chi'_c(H\Box C_{2m+1}) \geq s + 1/\lfloor\lambda(1-1/s)\rfloor$, where λ is the smallest maximum length of a cycle in a basis of the cycle space of an orientation of H. When $H = C_{2k+1}$ and m is large, the lower bound is sharp. In particular, if $m \geq 3k+1$, then $\chi'_c(C_{2k+1}\Box C_{2m+1}) = 4 + 1/\lceil 3k/2\rceil$, independent of m.

II: *Proper path-factors and interval edge-coloring of* $(3,4)$*-biregular bigraphs* (joint work with Armen S. Asratian, Linköping University; Carl Johan Casselgren, Umeå University; and Jennifer Vandenbussche, University of Illinois).

An *interval coloring* of a graph G is a proper coloring of $E(G)$ by positive integers such that the colors on the edges incident to any vertex are consecutive. A $(3,4)$*-biregular bigraph* is a bipartite graph in which each vertex of one part has degree 3 and each vertex of the other has degree 4; it is unknown whether these all have interval colorings. We prove that G has an interval coloring using 6 colors when G is a $(3,4)$-biregular bigraph having a spanning subgraph whose components are paths with endpoints at 3-valent vertices and lengths in $\{2,4,6,8\}$. We provide sufficient conditions for the existence of such a subgraph.

III: *Parity and strong parity edge-colorings of graphs* (joint work with David P. Bunde, Kevin Milans, and Hehui Wu, University of Illinois).

A *parity walk* in an edge-coloring of a graph is a walk along which each color is used an even number of times. We introduce two parameters. Let $p(G)$ be the least number of colors in a *parity edge-coloring* of G (a coloring having no parity path). Let $\hat{p}(G)$ be the least number of colors in a *strong parity edge-coloring* of G (a coloring having no open parity walk). Always $\hat{p}(G) \geq p(G) \geq \chi'(G)$.

Always $p(G) \geq \lceil \lg n(G) \rceil$, with equality for paths and even cycles. When n is odd, $p(C_n) = \hat{p}(C_n) = 1 + \lceil \lg n \rceil$. Although $p(G)$ and $\hat{p}(G)$ may differ, equality is conjectured to hold for all bipartite graphs. The main result that $\hat{p}(K_n) = 2^{\lceil \lg n \rceil} - 1$ generalizes a special case of Yuzvinsky's Theorem; the conjecture that $p(K_n) = \hat{p}(K_n)$ holds for $n \leq 16$. We mention further results and many open problems.

THE NUMBER OF INDEPENDENT SETS INTERSECTING THE SET OF LEAVES IN TREES

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A subset $S \subseteq V(G)$ is independent if no two vertices of S are adjacent in G. The number of independent sets in G is denoted by $NI(G)$. In the chemical literature the graph parameter $NI(G)$ is referred to as the Merrifield-Simmons index. The study of the number $NI(G)$ of independent sets in a graph was initiated in [3]. The literature includes many papers dealing with the theory of counting of independent sets in graphs, see [1, 2, 3, 4].

Let T be a tree. For $x \in V(T)$ denote by $L(x)$ the set of leaves attached to the vertex x. The vertex $x \in V(T)$ with $L(x) \neq \emptyset$ is called a *support vertex*. The set of all support vertices in T we denote by $S(T)$ and the set of leaves in T we denote by L. We consider independent sets intersecting the set of leaves. In particular we study independent sets S of T such that for every $x \in S(T)$, $S \cap L(x) \neq \emptyset$ i.e., S contains for each support vertex at least one leaf. Next we calculate the number of all independent sets which contain L as a subset. In each case we characterize extremal trees.

Keywords: independent set, counting, Fibonacci numbers, trees, structural characterizations.

AMS Subject Classification: 05C20.

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