11th WORKSHOP ON GRAPH THEORY

COLOURINGS, INDEPENDENCE AND DOMINATION

CID

Karpacz 2005, September 18-23

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ON THE NORMALITY OF CAYLEY GRAPHS

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Let G be a finite group, and let $1_G \notin S \subseteq G$. The *Cayley di-graph* $\Gamma = Cay(G,S)$ of G relative to S is the di-graph with vertex set G such that, for $x, y \in G$, the pair (x, y) is an arc if and only if $yx^{-1} \in S$. Further, if $S = S^{-1} := \{s^{-1} | s \in S\}$, then Γ is undirected. Γ is connected if and only if $G = \langle s \rangle$.

A Cayley (di)graph $\Gamma = Cay(G,S)$ is called *normal* if the right regular representation of G is a normal subgroup of the automorphism group of Γ.

The concept of normality of Cayley (di)graphs is known to be important for the study of arc-transitive graphs and half-transitive graphs. Given a finite group G, a natural problem is to determine all normal or non-normal Cayley (di)graphs of G. This problem is very difficult and is solved only for the cyclic groups of prime order by Alspach and the groups of order twice a prime by Du, Wang and Xu.

Du, Wang and Xu characterized disconnected Cayley graphs. Therefore the main work to determine the normality of Cayley graphs is to determine the normality of connected Cayley graphs.

In this paper we first determine all normal undirected Cayley graphs of Dihedral groups with valency four. Moreover, we give a complete classification for arc-transitive Cayley graphs of valency five on finite Abelian groups.

NONREPETITIVE COLORINGS OF GRAPHS

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Let $k \geqslant 2$ be a fixed integer. A coloring f of the vertices of a graph G is *k-repetitive* if there is $n \geqslant 1$ and a simple path $v_1 v_2 ... v_{kn}$ of G such that $f(v_i) = f(v_j)$ whenever $i - j$ is divisible by n. Otherwise f is called *k-nonrepetitive*. The minimum number of colors needed for a k-nonrepetitive coloring of G is denoted by $\pi_k(G)$. Notice that any 2-nonrepetitive coloring must be proper in the usual sense, while this is not necessarily the case for $k \geqslant 3$.

By the 1906 theorem of Thue [6] $\pi_2(G) \leqslant 3$ and $\pi_3(G) \leqslant 2$ if G is a simple path of any length. Let $\pi_k(d)$ denote the supremum of $\pi_k(G)$, where G ranges over all graphs with $\Delta(G) \leqslant d$. A simple extension of probabilistic arguments from [2] (for $k = 2$) shows that there are absolute positive constants c_1 and c_2 such that

$$c_1 \frac{d^{k/(k-1)}}{(\log d)^{1/(k-1)}} \leqslant \pi_k(d) \leqslant c_2 d^{k/(k-1)}.$$

Moreover, one can show that for each d there exists a sufficiently large $k = k(d)$ such that $\pi_k(d) \leqslant d + 1$. On the other hand, any $\lfloor d/2 \rfloor$-coloring of a d-regular graph of girth at least $2k + 1$ is k-repetitive. The maximum number $t(d)$ such that for each k there is a d-regular graph G with $\pi_k(G) > t(d)$ is not known for $d \geqslant 3$.

Kündgen and Pelsmajer [4] and Barát and Varjú [3] proved independently that $\pi_2(G)$ is bounded for graphs of bounded treewidth. By the result of Robertson and Seymour [5] it follows that if H is any fixed planar graph then $\pi_k(G)$ is bounded for graphs not containing H as a minor. However, it is still not known whether there are some constants k and c such that $\pi_k(G) \leqslant c$ for any planar graph G. The least possible constant c for which this could hold (with possibly huge k) is $c = 4$.

In a weaker version of the problem we ask for nonrepetitive colorings of subdivided graphs. By the result of Thue every graph has a (sufficiently large) subdivision which is nonrepetitively 5-colorable (for any $k \geqslant 2$). Clearly this cannot happen for all graphs if we restrict the number of vertices added to an edge. For instance, any c-coloring of the complete graph K_n, with each edge subdivided by at most r vertices, is 2-repetitive if $c < \log_r \log_2(n/r)$. The question if there are constants c, k, and r such that each planar graph G has an r-restricted subdivision S with $\pi_k(S) \leqslant c$, is open.

There are many interesting connections of this area to other graph coloring topics. Let $s(G)$ be the *star chromatic number* of a graph G, that is, the least number of colors in a proper coloring of the vertices of G, with additional property that every two color classes induce a star forest. It is not hard to see that $\pi_2(G) \geqslant s(G)$ for any graph G. Hence, by the results of Albertson et al. [1] it follows that there are planar graphs with $\pi_2(G) \geqslant 10$, and for each t there are graphs of treewidth t with $\pi_2(G) \geqslant \binom{t+1}{2}$.

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ANTIMAGIC LABELINGS OF THE TREES

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A graph $G = (V, E)$ is (a, d)-*edge-antimagic total* if there exists a bijective function $f : V(G) \cup E(G) \to \{1, 2, \ldots, |V(G)| + |E(G)|\}$ such that the edge-weights $w(uv) = f(u)+f(v)+f(uv), uv \in E(G)$, form an arithmetic progression with initial term a and common difference d. Such a labeling is called *super* if the smallest possible labels appear on the vertices. In this paper we study super (a, d)-edge-antimagic properties for paths and for a special class of trees called path-like trees.

ON THE COST CHROMATIC NUMBER OF GRAPHS

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In a graph, by definition, the weight of a (proper) coloring with positive integers is the sum of the colors. The *cost chromatic number* or the *chromatic sum* is the minimum weight of all the proper colorings. The minimum number of colors in a coloring of minimum weight is the *strength* of the graph. We derive general upper bounds for the strength in terms of a new parameter of representations by edge intersections of hypergraphs.

THE GAME OF ARBORICITY

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We consider the following graph coloring game. Ann and Ben alternately color the edges of a graph G using a fixed set of colors C. The only restriction they both have to respect is that no monochromatic cycle may be created. Ann wants to accomplish a coloring of the whole graph G, while Ben aims to achieve a partial coloring that would not be extendable without violating the acyclicity condition. The minimum size of C guaranteeing a win for Ann is the *game arboricity* of G, which we denote by $A_g(G)$. Clearly, $A_g(G)$ is at least $A(G)$ — the usual arboricity of a graph G, that is, the minimum number of forests needed to cover the edges of G.

Let $L(G)$ denote the minimum of the largest vertex outdegree taken over all orientations of a graph G. Our main result [2] asserts that $A_g(G) \leqslant 3L(G)$ for any graph G. We achieve it by a suitable directed-edge version of the *activation strategy*. Clearly, $L(G) \leqslant A(G)$, so we have also $A_g(G) \leqslant 3A(G)$ for any graph G. On the other hand, for any k we construct a graph G of arboricity k such that $A_g(G) \geqslant 2k-2$. We also prove that $A_g(G) = 2$ for any 2-degenerate graph G, which supports a conjecture that perhaps $A_g(G) \leqslant d$, for each $d \geqslant 1$ and any d-degenerate graph G. If true this would imply that the lower bound is almost exact.

The arboricity game is a variant of the well-studied vertex coloring game, introduced independently by Brams [5] and Bodlaender [3]. In the original game the players color the vertices of a graph G so that no monochromatic edge may be created. The related parameter is called the game chromatic number, denoted by $\chi_g(G)$. The most challenging problem here is to determine the game chromatic number of planar graphs. At present it is known that 17 colors suffice [10] for any planar graph, and that there are planar graphs demanding at least 8 colors [7]. The topic is related to arrangability, acyclic colorings, graph orderings and the related parameters (cf. [4], [7], [8]).

Our results show that $A_g(G)$ behaves more tamely than $\chi_g(G)$. In fact, the difference between chromatic number $\chi(G)$ and game chromatic number $\chi_g(G)$ can be arbitrarily large already for bipartite graphs. Also some questions which are hard for the vertex game can be easily answered (at least for some classes of graphs) using our results. For instance, in [9] Zhu asked whether the fact that Ann wins the vertex game with k colors on a graph G implies that she wins with $k+1$ colors, too. At first glance the question looks like a joke — the more colors, the better for Ann. However, despite some efforts, no proof was supplied so far. For arboricity game Zhu's question has a positive answer for 2-degenerate graphs. Also a weaker version of the problem, which asks for a function $f(k) > k$ such that Ann wins with $f(k)$ colors provided she wins with k colors, is easily answered by our theorem, as we may take $f(k) = 3k$. A nice challenging problem would be to determine the game arboricity of planar graphs. The above result implies that $A_g(G) \leqslant 9$ for any planar graph G which does not seem to be the best possible bound.

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WEIGHT CHOOSABILITY AND COMBINATORIAL NULLSTELLENSATZ

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Let S be a subset of the integers. We say G is S-*weight colorable* if there is an edge weighting $w : E \to S$ such that for any two adjacent vertices u, v of G the sum of weights of the edges incident to u is different than the sum of weights of the edges incident to v.

Conjecture 1. (Karoński, Łuczak, Thomason [2])
Each connected graph with more than one edge is $\{1, 2, 3\}$-*weight colorable.*

Recently Addario-Berry et al. [1] proved that any such graph is $\{1, 2, ..., 16\}$-weight colorable, and that for any fixed $p \in (0, 1)$ the random graph $G_{n,p}$ is asymptotically almost surely 2-weight colorable. We attempt to attack the conjecture by applying Alon-Tarsi Combinatorial Nullstellensatz. Roughly, we assign to a given graph G a polynomial in $m = |E(G)|$ variables $f(x_1, ..., x_m)$ which encodes our problem, and try to prove that there must be a nonvanishing monomial in f each of whose exponents is at most 2. If this would be true the conjecture would follow in the following stronger sense. Let $S_1, ..., S_m$ be a collection of integer subsets assigned to the edges of G. We say that G is d-*weight choosable* if for any such assignment with $|S_1| = ... = |S_m| = d$, there is an edge weighting $w : E \to \mathbb{Z}$ such that $w(e_i) \in S_i$, and for any two adjacent vertices u, v the sum of weights of the edges incident to u is different than the sum of weights of the edges incident to v.

Conjecture 2.
Every connected graph with at least two edges is 3-*weight choosable.*

Maybe this conjecture is too optimistic, as we even do not know if there is any finite bound. But why not to state optimistic conjectures?

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SATISFACTORY GRAPH PARTITIONS AND THEIR GENERALIZATIONS

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A *satisfactory partition* of a graph $G = (V, E)$ is a vertex bipartition $V_1 \cup V_2 = V$ into nonempty parts such that each vertex $v \in V_i$ is adjacent to at least as many vertices in V_i as in V_{3-i} $(i = 1, 2)$. In the talk we survey results and open problems concerning the existence and search of satisfactory partitions and some related generalizations. The latter include partitions into more than two classes, weighted variants, more general conditions on vertex degrees, and restrictions on the class sizes of the partitions. Restricted graph classes and approximation algorithms will also be considered.

CHROMATIC POLYNOMIALS FOR RECURSIVELY DEFINED FAMILIES OF GRAPHS

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The *chromatic polynomial* $P(G, \lambda)$ of a graph G in the variable λ counts for positive integers λ the proper vertex λ-colourings of G.

In this paper we give an explicite formula for the chromatic polynomial of some strip graphs with free boundary conditions and cylindrical boundary conditions. Moreover we study the location of chromatic zeros for some families of such graphs. A relation of chromatic zeros with some phenomena in statistical mechanics will be presented.

AMS Subject Classification: 05C15.

ON ACYCLIC COLOURINGS OF SOME CLASSES OF GRAPHS

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Following [4], let $\mathcal{P}_1$, $\mathcal{P}_2$, $\ldots$, $\mathcal{P}_k$ be hereditary properties of graphs. A $(\mathcal{P}_1, \mathcal{P}_2, \ldots, \mathcal{P}_k)$-*colouring* of a graph G is a mapping f from the set of vertices of G to a set of k colours such that for every colour i, the subgraph induced by the i-coloured vertices has property $\mathcal{P}_i$. Such a colouring is called *acyclic*, if for every two distinct colours i and j, the subgraph induced by all the edges linking an i-coloured vertex and a j-coloured vertex is acyclic.

The notion of acyclic colourings was introduced by Grűnbaum [5], who asked about planar graphs. In [3] Borodin solved this problem showing that every planar graph can be acyclically 5-coloured. Sopena, Boiron and Vignal studied acyclic $(\mathcal{P}_1, \mathcal{P}_2, \ldots, \mathcal{P}_k)$-colourings (also called acyclic improper colourings) of planar and outerplanar graphs, see [1], and graphs with bounded degree, see [2].

In this paper we continue their work, considering acyclic $(\mathcal{P}_1, \mathcal{P}_2, \ldots, \mathcal{P}_k)$-colourings of some classes of graphs. Among others, we prove some new positive and negative results for outerplanar graphs.

Keywords: graph, acyclic colouring, graph property.
AMS Subject Classification: 05C15.

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ON LIST CHROMATIC NUMBER OF CARTESIAN PRODUCT OF TWO GRAPHS

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Given a list $L(v)$ of colours for each vertex v of graph G, we say that a vertex colouring is *acceptable* if every vertex is coloured with a colour on its list and no two adjacent vertices is assigned with the same colour. The *list chromatic number* of graph G, denoted $\chi_l(G)$ is minimum r, which satisfies: if every list has at least r members then there is an acceptable colouring. We deal with problem formulated by S. Jendrol' and M. Borowiecki

> Let $\chi_l(G)$ denotes list chromatic number of G and let $G \times H$ be the Cartesian product of graphs G and H. Does there exist an absolute constant c such that $\chi_l(G \times H) \leqslant \max\{\chi_l(G), \chi_l(H)\} + c$. If answer is YES, how big is c ? Does $c = 1$?

We prove it in case if one graph is a tree T, i.e. for list chromatic number holds $\chi_l(T \times H) \leqslant \max\{\chi_l(T), \chi_l(G)\} + 1$. We present an counterexample to hypothesis in arbitrary case. For arbitrary graphs G and H we prove weaked theorem $\chi_l(G \times H) \leqslant \min\{\chi_l(G) + col(H), \chi_l(H) + col(G)\} - 1$. We also bound list chromatic number of $G \times G$ for arbitrary graph G asymptotically as follows $\chi_l(G \times G) \leqslant \triangle(G) + o(\triangle(G))$ for $\triangle(G) \to \infty$.

GENERALIZED ON-LINE COLORING

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In this talk the concepts from two intensively studied frameworks, i.e., *on-line coloring* and *generalized coloring* of graphs are combined to define and investigate generalized on-line colorings.

Among many generalizations of classical graph coloring problem we are especially interested in on-line $\mathcal{P}$-colorings where $\mathcal{P}$ is some additive hereditary property of graphs (see [1]) for a survey of hereditary properties). On-line $\mathcal{P}$-coloring turned out to be advantageous in analysis of various on-line resource management problems [2].

On-line coloring can be viewed as a game of two adversaries called *Presenter* and *Painter*. Painter (on-line algorithm) does not know the structure of a graph to be colored. Presenter reveals subsequent vertices of graph G in some order $(v_1, \ldots, v_n)$ which is unknown to Painter. Vertex v_i is presented together with edges $E_i \subseteq E(G)$ to its already presented neighbors. When vertex v_i is presented, Painter has to irrevocably assign $c(v_i)$ - one of the permissible colors and it has to be done before the next vertex is presented. The goal of Painter, to use as few colors as possible, is opposed to the strategy of Presenter which aims at finding the vertex ordering that forces Painter to use as much colors as possible. One of the best known heuristics for on-line graph coloring is the First-Fit algorithm, which assigns to each vertex as small color as possible (see [3],[4]) for surveys on on-line coloring).

We define a family of greedy on-line $\mathcal{P}$-coloring algorithms, give some lower and upper bounds for on-line $\mathcal{P}$-chromatic number and prove that for some classes of graphs our algorithms are best possible. A finite basis theorem for the family of on-line First-Fit $(\mathcal{P}, k)$-colorable graphs is given.

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ON EFFICIENT DOMINATING SETS IN TREES

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and

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A vertex *dominates* both itself and each of its neighbors. A vertex set S is *efficient dominating set* in a graph G if each vertex of G is dominated by exactly one member of S. Trees on n vertices with largest possible number of efficient dominating sets are characterized and enumerated. In particular, in the set of those trees for $n = 7b + 2i$ with $i = -1, 0, \ldots, 5$, given any prescribed tree T on b vertices where $b \geqslant 5$, there is a member which includes T as an induced subtree.

Keywords: tree, efficient dominating set, maximizing cardinality, structure.
AMS Subject Classification: 05C05, 05C69, 05C35, 05C75, 05A15.

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THE OUTER-CONNECTED DOMINATION NUMBER OF A GRAPH

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For a given graph $G = (V, E)$, a set $D \subseteq V(G)$ is said to be an *outer-connected dominating set* if D is dominating and the graph $G - D$ is connected. The *outer-connected domination number* of a graph G, denoted by $\widetilde{\gamma}_c(G)$, is the cardinality of a minimum outer-connected dominating set of G. We study several properties of outer-connected dominating sets and give some bounds on the outer-connected domination number of a graph. We also show that the decision problem for the outer-connected domination number of a graph G is NP-complete even for bipartite graphs.

Keywords: outer-connected domination number, domination number.
AMS Subject Classification: 05C05, 05C69.

JOINS OF ADDITIVE HEREDITARY PROPERTIES OF GRAPHS

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Let $\mathbb{L}^a$ denote a set of additive hereditary graph properties. It is a known fact that a partially ordered set $(\mathbb{L}^a, \subseteq)$ is a complete distributive lattice. In the paper we decide when a join of two additive hereditary graph properties in $(\mathbb{L}^a, \subseteq)$ has a finite or infinite family of forbidden subgraphs. Moreover, we show that the analysed class of graph properties is disjoint with the set of reducible over $\mathbb{L}^a$ properties (it was previously observed using an other method in [2]). That means, each consequence of the paper is not included in the only known result of this type obtained by A. Berger [1] and stated that for any additive hereditary reducible property of graphs the family of forbidden subgraphs is infinite.

Keywords: hereditary property, lattice of additive hereditary graph properties.

AMS Subject Classification: 05C75, 05C15, 05C35.

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THE UPPER DOMINATION RAMSEY NUMBERS

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The *upper domination Ramsey number* $u(m, n)$ is the smallest integer p such that in every 2-coloring the edges of K_p with color red (R) and blue (B), $\Gamma(B) \geqslant m$ or $\Gamma(R) \geqslant n$, where $\Gamma(G)$ is the maximum cardinality of a minimal dominating set of a graph G. Up to now, there have been known only few exact values for such numbers. We present new bounds for $u(4, 4)$.

Keywords: edge coloring, upper domination Ramsey number.
AMS Subject Classification: 05C15, 05C55, 05C69.

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ON A GAME OF SOKOBAN TYPE

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A digraph is called *irregular* if its different vertices have different degree pairs. An irregular digraph with fixed number of vertices is the *largest* (the *smallest*) if it has the greatest (the smallest) possible number of arcs. The number of arcs of the largest (the smallest) irregular digraph is given in DM 236 (2001) 263–272.

In this talk a game of Sokoban type is presented. Any win strategy for this game gives a construction of a sequence of irregular digraphs in which the first element is the largest digraph, any successive digraph is obtained from previous one by the deletion of one arc and the last element is the smallest digraph.

Keywords: irregular digraph, degree pair.
AMS Subject Classification: 05C.

ON LINEAR ARBORICITY OF CUBIC GRAPHS

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A *linear-k-forest* is a graph whose connected components are chordless paths of length at most k. The *linear-k-arboricity* of a graph G (denoted $la_k(G)$) is the minimum number of linear-k-forests which partition $E(G)$ (when $k = |V(G)|-1$ a linear-k-forest is merely a *linear forest*, $la(G)$ is then the minimum number of linear forests partitioning $E(G)$, a *linear partition*). When such a partition of $E(G)$ has been imposed, it is said that G has been *factored* into linear-k-forests.

We consider linear partitions of cubic graphs (for which it is known that $la(G) = 2$ (Akiyama, Chvatal)), and we show that paths of length 3 play a central role.

An *odd linear forest* is a linear forest in which each component is a path of odd length. Aldred and Wormald proved that a cubic graph G can be factored into two odd linear forests if and only if G is 3-edge coloured. Let G be a 3-edge coloured cubic graph, we show that for any two colours there exists a strong matching M such that every bicoloured cycle of G contains at least one edge in M and we study some consequenses of this result.

Let $L = (L_1, L_2)$ be a linear partition of a cubic graph. Every vertex v is either end-vertex of a path of L_1 or end-vertex of a path of L_2. We denote by $e_L(v)$ the edge of this path incident to v. Two linear partitions $L = (L_1, L_2)$ and $L' = (L'_1, L'_2)$ are *compatible* if for every vertex v, $e_L(v) \neq e_{L'}(v)$. Fouquet (1991) conjectured that there exist two compatible linear partitions in a cubic graph. We give some partial results reinforcing this conjecture.

EQUITABLE LIST COLORING OF GRAPHS

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We consider the problem of graph coloring introduced by Kostochka et al. in 2003 [1]. Given lists of available colors assigned to each vertex of an n-vertex graph G, a *list coloring* is a proper coloring such that the color on each vertex is chosen from its list. If the lists all have size k then list coloring is equitable if each colors appears on at most $\lceil n/k \rceil$ vertices. We say that a graph is *equitably k-choosable* if such coloring exists for all list assignments L such that $|L(v)| = k$ for all $v \in V(G)$.

Kostochka et al. in [1] formulated following conjecture

Conjecture.
Every graph G is equitably k-choosable whenever $k \geqslant \Delta(G) + 1$.

We proved this conjecture for split graphs and some complete r-partite graphs.

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COUNTING CHROMATIC STRUCTURES BY MATRIX NETWORKS

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Subject of our research is enumerative combinatorics of finite chromatic structures, i.e. colorings of finite sets satisfying given structural conditions (predicats). The general enumerative problem we consider is following: what is the number of chromatic structures (i.e. color images) of given type? The type is defined by parameters of underliing sets and by given family of predicats. Such formulation collapses most all famous enumerative combinatorial problems about colorings, including, in particular, colorings of graphs and hypergraphs.

We propose an original method for solution of above formulated general problem, namely the *multidimensional matrix network method*.

The multidimensional matrix network, or simply, matrix network is by definition a family of multi-index matrices connected into entire net by means of a collection of matrix multiplications. After performing the multiplications, some indices may remain free, unutilized, and some others closed. Correspondingly, we may distinguish two type networks: open or closed. The open networks are itself multidimensional matrices and may be used to form some other networks. But the closed networks represent some numbers or functions obtained as result of network multiplications and hence expressing some integral quantities depending on network structure. These quantities are the combinatorial numbers (or functions) for the given type chromatic structures.

SOME NEWS ABOUT INDEPENDENCE

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A known upper bound for the independence number of a regular graph is realized by a continuous optimization problem. Considering the necessary optimality conditions the tightness of the bound is investigated.

RECENT RESULTS ON TOTAL DOMINATION IN GRAPHS

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A set S of vertices in a graph G without isolated vertices is a *total dominating set* of G if every vertex of G is adjacent to a vertex in S. The *total domination number* of G is the minimum cardinality of a total dominating set in G. In this talk, we discuss recent results on total domination in graphs.

AMS Subject Classification: 05C69.

COMBINATORIAL LEMMAS FOR POLYHEDRONS I

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We formulate general boundary conditions for a labelling of vertices of a triangulation of a polyhedron by vectors to assure the existence of a balanced simplex. The condition is not for each vertex separately, but for a set of vertices of each boundary simplex. This allows us to formulate a theorem, which is more general then Sperner's lemma and theorems of Shapley, Ichiishi, Idzik, Junosza-Szaniawski. Our results are related to the theorem of van der Laan, Talman and Yang. A generalization of Poincaré theorem can be derived.

Keywords: labelling, pseudomanifold, simplicial complex, Sperner lemma.
AMS Subject Classification: 05B30, 47H10, 52A20, 54H25.

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COLORING DENSE PLANAR GRAPHS

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The paper presents an algorithm of coloring cycles of a planar graph with vertices of at least a degree of five. The introduced notions of cycles, edges' cyclic numbers, and tiers and layers are crucial components of the algorithm. A planar graph is decomposed into layers of cycles. The algorithm demonstrates that at most three colors are sufficient for a layer and four colors are sufficient for the graph.

ON TOTAL IRREGULAR LABELLINGS OF GRAPHS

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For a simple graph $G = (V, E)$ with vertex set V and edge set E, a labelling $\lambda : V \cup E \to \{1, 2, \dots, k\}$ is called a *total k-labelling*. The weight of an edge xy under a total k-labelling λ is defined as

$$wt(xy) = \lambda(x) + \lambda(xy) + \lambda(y)\,.$$

A total k-labelling is defined to be a *total edge-irregular k-labelling* of a graph G if, for every two different edges e and f of G,

$$wt(e) \neq wt(f)\,.$$

The minimum k for which a graph G has a total edge-irregular k-labelling is called the *total edge irregularity strength* of G, $tes(G)$.

We present a survey on results concerning both above mentioned graph characteristics.

THE CIRCULAR CHROMATIC INDEX OF GRAPHS OF HIGH GIRTH

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Colorings of graphs form a prominent topic in graph theory. Several relaxations of usual colorings have been introduced and intesively studied. In this talk, we focus on circular colorings of line graphs. A *proper circular k-edge coloring*, for real $k \geqslant 1$, is a coloring by real numbers from the interval $[0, k)$ such that the difference modulo k of the colors c_1 nad c_2 assigned to incident edges is at least one, i.e., $1 \leqslant |c_1 - c_2| \leqslant k - 1$.

A classical theorem of Vizing states that the edges of every graph G with maximum degree Δ can be colored by at most $\Delta + 1$ colors so that no two incident edges have the same color, i.e., the chromatic index of G is at most $\Delta + 1$. We show that for every $\varepsilon > 0$ there exists g such that the circular chromatic index of a graph G with maximum degree Δ whose girth is at least g does not exceed $\Delta + \varepsilon$. Note that the index must be at least Δ because the line graph of such graph G contains a clique of order Δ.

Our research is motivated by a conjecture of Jaeger and Swart 1979 that high girth cubic graphs have chromatic index three, which was disproved by Kochol 1996. Our results imply that the conjecture is true when relaxed to circular colorings: the circular chromatic index of high girth cubic graphs is close to three.

One of the ingredients of our proof are recent result on systems of independent representatives and hypergraph matchability of by Aharoni, Haxell, Meshulam and others, which we also briefly survey during the talk.

CONSTRUCTIONS VIA HAMILTONIAN THEOREMS

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Demetrovics, Sali and the present author constructed a decomposition of the family of all k-element subsets of an n-element set into disjoint pairs (A, B) $(A \cap B = \emptyset, |A| = |B| = k)$ where two such pairs are relatively far from each other in some sense. The paper invented a proof method using a Hamiltonian type theorem. This was sharpened in a joint paper with Enomoto where the result became of coding type. The present paper gives a generalization of this tool, hopefully extending the power of the method. Problems where the method could be also used are shown. Moreover open problems are listed which are related to the Hamiltonian theory. In these problems a cyclic permutation is to be found when certain restrictions are given by a family of k-element subsets. An interesting generalization of Baranyai's theorem is posed as an open problem.

THE CIRCULAR CHROMATIC INDEX

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A (k,d)-*edge coloring* ($k,d \in \mathbb{N}$, $k \geqslant 2d$) of a graph G is an assignment c of colors $\{0,1,\ldots,k-1\}$ to the edges of G such that $d \leqslant |c(e_i) - c(e_j)| \leqslant k-d$ whenever two edges e_i and e_j are adjacent. The *circular chromatic index* $\chi'_c(G)$ is defined by $\chi'_c(G) = \inf\{k/d : G \text{ has a } (k,d)\text{-edge coloring}\}$. We prove several properties of $\chi'_c(G)$ and determine exact values for some classes of graphs.

Keywords: star chromatic number, circular chromatic number, circular chromatic index, edge coloring.
AMS Subject Classification: 05C15.

DOMATIC NUMBER OF GRAPH PRODUCTS

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A partition of $V(G)$, all of whose classes are dominating sets in G, is called a *domatic partition* of G. The maximum number of classes of a domatic partition of G is called the *domatic number* of G. The concept of a domatic number was introduced in [1]. More interesting results on domatically full graphs, domatically critical, domatically cocritical graphs and other domatic numbers can be found in [3], [4], [5], [6], [7].

We explore the bounds the domatic number of the cartesian product, the strong product and the corona of two graphs. The join of two graphs and its generalization also is studied. Motivation of this problem comes from [2], where the domatic number of the cartesian product of two paths was established.

Keywords: domatic number, cartesian product of graphs, strong product of graphs, join of graphs, corona of graphs.
AMS Subject Classification: 05C69.

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EXTREMAL PROBLEMS ON H-LINKED GRAPHS

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Let H be a graph. An H-*subdivision* in a graph G is a pair of mappings $f : V(H) \to V(G)$ and g: $E(H)$ into the set of paths in G such that:
(a) $f(u) \neq f(v)$ for all distinct $u, v \in V(H)$;
(b) for every $uv \in E(H)$, $g(uv)$ is an $f(u)f(v)$-path in G, and distinct edges map into internally disjoint paths in G.
A graph G is H-*linked* if every injective mapping $f : V(H) \to V(G)$ can be extended to an H-subdivision in G.

The notion of an H-linked graph is a common generalization of the notions of k-linked graphs, k-ordered graphs and k-connected graphs. For example, a graph G on at least $k+1$ vertices is k-connected if and only if G is S_k-linked, where S_k is the star with k rays. A graph G is k-ordered if and only if G is C_k-linked, where C_k is the cycle with k edges. A graph G is k-linked if and only if G is M_k-linked, where M_k is the matching with k edges.

The idea of H-linked graphs originated with Jung more than thirty years ago, but had not been significantly developed until recently, when the concept was considered by several authors.

The aim of the talk is to survey recent results on extremal problems on H-linked graphs with some restrictions on the structure of H.

LOWER BOUND ON THE WEAKLY CONNECTED DOMINATION NUMBER OF A TREE

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We prove that the weakly connected domination number of every tree T on $n \geqslant 3$ vertices and with n_1 end-vertices satisfies inequality

$$\gamma_w(T) \geqslant \frac{1}{2}(n(T) + 1 - n_1(T))$$

and we characterize the extremal trees.

Keywords: weakly connected domination number, tree.
AMS Subject Classification: 05C05, 05C69.

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ON THE NUMBER OF MAXIMAL INDEPENDENT SETS IN 3-UNIFORM HYPERGRAPHS

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In 1965 Moon and Moser answered the following question raised by Erdős and Moser: "What is the maximum number $f_2(n)$ of maximal independent sets possible in a graph with n vertices ?" They proved that $f_2(n) = 3^{n/3}$, for n divisible by 3 (for n not divisible by 3, $f_2(n)$ differs from $3^{n/3}$ by a constant factor). We consider an analogous question for 3-uniform hypergraphs: "What is the maximum number $f_3(n)$ of maximal independent sets possible in a 3-uniform hypergraph with n vertices ?" Tomescu gave a construction of a family of 3-uniform hypergraphs with n vertices which, for n divisible by 5, have $10^{n/5}$ maximal independent sets. Consequently, $f_3(n) \geqslant 10^{n/5} \approx 1.5848..^n$, for n divisible by 5. When n is not divisible by 5, the lower bound differs from the bound written above by a constant factor. We show that $f_3(n) \leqslant 1.6701..^n$. In fact we prove a bit stronger result that $f_{\leqslant 3}(n) \leqslant 1.6701..^n$, where $f_{\leqslant 3}(n)$ is the maximum number of maximal independent sets in a hypergraph with n vertices in which the cardinality of every edge is at most 3.

CYCLES IN CLAW-FREE GRAPHS

Tomasz Łuczak
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We present some old and new theorems on the cyclic structure of claw-free graphs, concentrating on recent results obtained jointly with Ronald J. Gould and Florian Pfender.

PATH DECOMPOSITIONS OF A COMPLETE MULTIDIGRAPH

Mariusz Meszka and Zdzisław Skupień
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The general conjecture says that the complete n-vertex multidigraph ${}^{\lambda}\mathcal{D}K_n$ (ie. the multidigraph obtained by replacing each arc of the complete digraph $\mathcal{D}K_n$ of order n by λ arcs) is decomposable into directed paths of arbitrarily prescribed lengths provided that the lengths sum up to the size $\lambda\, n\,(n-1)$ of ${}^{\lambda}\mathcal{D}K_n$, unless all paths are hamiltonian and either $n = 3$ and λ is odd or $n = 5$ and $\lambda = 1$.

Supporting results for the conjecture will be presented, especially in the case when all required paths are to be nonhamiltonian.

SOME PROPERTIES OF $\mathcal{P}$-CONNECTED $\mathcal{P}$-DOMINATION NUMBERS OF GRAPHS

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Let $\mathcal{P}$ be an induced addtitive hereditary property of graphs. Let $\mathcal{F}(\mathcal{P})$ be a set of forbidden subgraphs for $\mathcal{P}$, i.e. $\mathcal{F}(\mathcal{P}) = \{H \in \mathcal{I} : H \notin \mathcal{P}$ but $(H - v) \in \mathcal{P}$ for any $v \in V(H)\}$.

Let $G = (V, E)$ be a graph. For a vertex $v \in V$, by $N_{\mathcal{P}}(v)$ we denote the $\mathcal{P}$-neighbourhood of v, i.e. $N_{\mathcal{P}}(v) = \{u \in V : u$ is $\mathcal{P}$-adjacent to $v\}$. Two vertices u and v are $\mathcal{P}$-adjacent if there is an induced subgraph H' of G and H' is a graph isomorphic to some $H \in \mathcal{F}(\mathcal{P})$ (shortly $H' \in \mathcal{F}(\mathcal{P})$ containing u and v. Let $e \in E(G)$. If there is no subgraph H of G, $H \in \mathcal{F}(\mathcal{P})$ such that e is contained in H, then e is called $\mathcal{P}$-*isolated edge*.

A set D is said to be $\mathcal{P}$-*dominating* in G if for each $v \in V - D$, $N_{\mathcal{P}}(v) \cap D \neq \emptyset$.

A set D is said to be *strong* $\mathcal{P}$-*dominating* in G if for each $v \in V - D$, there is an induced subgraph H', $H' \in \mathcal{F}(\mathcal{P})$ containing v such that $V(H') - \{v\} \subseteq D$. A graph G is called a $\mathcal{P}$-*connected* if G has no $\mathcal{P}$-isolated edges and G is a connected graph.

A set D is said to be $\mathcal{P}$-*connected* $\mathcal{P}$-*dominating* ($\mathcal{P}$-*connected strong* $\mathcal{P}$-*dominating*) in G if D is $\mathcal{P}$-dominating (strong $\mathcal{P}$-dominating) and the subgraph induced by D is a $\mathcal{P}$-connected graph.

There are given some inequalities of corresponding domination numbers and also are given some some generalizations of well known theorems for graphs, namely Gallai type theorem generalizing Nieminen, Hedetniemi and Laskar theorems.

LATTICE OF ADDITIVE HEREDITARY PROPERTIES AND FORMAL CONCEPT ANALYSIS

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A graph property $\mathcal{P}$ is any nonempty proper isomorphism closed subclass of the class $\mathcal{I}$ of finite simple graphs. A graph property is said to be hereditary if it is closed under taking subgraphs and additive if it is closed under disjoint union of graphs. The lattice $\mathbb{L}^a$ of additive hereditary graph properties have been introduced and investigated in connection with generalized colourings of graphs. Some results on the structure of the lattice $\mathbb{L}^a$ can be easily generalized and proved for digraphs, hypergraphs, posets and other structures. The aim of our talk is to introduce additive hereditary properties as concepts of an appropriate context in the Formal Concept Analysis (FCA). The first paper on the Formal Concept Analysis was published by R. Wille in 1982. The method has been originally proposed as a mathematical model of data analysis and it is used as a tool for structured knowledge representation e.g. in the semantic web. FCA is based on a notion of a formal context, which is defined as a triple $(O, A, \models)$, where O-*objects* and A-*attributes* are non-empty classes/sets and $\models$ is a binary incidence relation between O and A. The *concepts* of a context $(O, A, \models)$ are ordered pairs (X, Y) where $X \subset O$ and $Y \subset A$ are closed sets wits respect to related closure operators on O and A, respectively. We will show that the lattice $\mathbb{L}^a$ of additive hereditary properties is in fact the complete algebraic lattice of formal concepts of the the context $(\mathcal{I}, -\mathcal{C}, \in)$ where the objects are finite graphs, i.e. $\mathcal{I}$, the attributes are the properties $-F$ - "do not contain a given finite connected graph F" and the incidence relation is simply $G \in -F$. Thus each graph property $\mathcal{P}$ has its *extent* $X = \mathcal{P} \subset \mathcal{I}$ and its *intent* $Y = \mathcal{F}$, the class of forbidden graphs for $\mathcal{P}$. In this context, we can use FCA to obtain deeper view to additive hereditary graph properties. For example, if we enlarge $\mathcal{I}$ in the given context to the class $\mathcal{I}^*$ of all simple (also infinite) graphs, we obtain an isomorphic lattice of additive graph properties of finite character. Our approach can be used analogousely to *object-system* and we will describe the ideas and reasons, which allows us to prove different types of general statements as e.g. Unique Factorization Theorems.

ON COLORING A COMMUTATIVE SEMIRINGS

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In 1988, Beck introduced the concept of coloring a commutative ring and determined the chromatic number of commutative rings which are finite colorable. A commutative ring R is considered as a simple graph G_R with the vertex set R and two different vertices x and y are adjacent if and only if x is a zero-divisor of y. In this paper, we discuss the problem of coloring a commutative semiring. We compute the chromatic number of a finite Boolean algebra. Moreover, we construct some new classes of commutative semirings using $\mathbb{Z}_n$, the set of all integers modulo n and compute their chromatic number.

Keywords: commutative semiring, coloring, chromatic number.

THE EXTENDABLE GRAPHS

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Let G be a simple graph and $N(u)$ denote the *open neighbourhood of* $u \in V(G)$. We say that a set $S \subseteq V(G)$ is:

(1) *nearly perfect dominating set* ([2]) if for every vertex $u \in V(G) - S$, $|N(u) \cap S| \geqslant 1$ or

(2) *perfect dominating* set ([1]) if for every vertex $u \in V(G) - S$, $|N(u) \cap S| = 1$.

A set $S \subseteq V(G)$ having some property $\mathcal{P}$ is called $\mathcal{P}$*-set*. We say that S is a 1*-maximal* $\mathcal{P}$*-set* if for every vertex $u \in V(G) - S$, the set $S \cup \{u\}$ does not have property $\mathcal{P}$. We define ${}_{\mathcal{P}}(G)$ to be the minimum cardinality of a 1-maximal $\mathcal{P}$-set in a graph G. The 1-maximal $\mathcal{P}$-set of the cardinality ${}_{\mathcal{P}}(G)$ we will call ${}_{\mathcal{P}}(G)$*-set*.

The connected graph G is said to be k*-extendable with respect to* $\mathcal{P}$*-set* (for short: k*-extendable*) if ${}_{\mathcal{P}}(G) \neq |V(G)|$ and every $\mathcal{P}$-set of size k in G is a proper subset of some ${}_{\mathcal{P}}(G)$-set. The maximum k such that G is k-extendable is the *extendability number* of a graph G and it is denoted by $E_{\mathcal{P}}(G)$.

We present some results concerning the relationship between extendability with respect to nearly perfect dominating set and extendability in the sense of perfect dominating set in special graphs.

Keywords: domination.
AMS Subject Classification: 05C69.

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RECENT ADVANCES IN SEVERAL AREAS OF DOMINATION IN GRAPHS

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A subset of verices D of a graph G is a *dominating set* for G if every vertex of G not in D is adjancent to one in D. The cardinality of any smallest dominating set in G is denoted by $\gamma(G)$ and called the *domination number* of G.

In this talk, we report on some recent results obtained with N. Ananchuen wnd with K. Kawaradayashi and A. Saito involving three different areas of domination in graphs.

Graph G is said to be γ-*edge-critical* if $\gamma(G + e) < \gamma(G)$ for each edge $e \notin E(G)$ and is said to be γ-*vertex-critical* if $\gamma(G - v) < \gamma(G)$ for each vertex $v \in V(G)$. The structure of both γ-edge-critical graphs and γ-vertex-critical graphs is not well understood, even in case when $\gamma(G) = 3$. We will present some new theorems involving matchings in both classes.

Reed conjectured in 1996 that if G is a cubic graph with n verices, then $\gamma(G) \leqslant \lceil \frac{|V(G)|}{3} \rceil$. We will close by presenting some new resaults pertaining to this conjecture.

DISTANCE PAIRED DOMINATION NUMBER OF A TREE

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We study a generalization of the paired domination number. A set $D \subseteq V(G)$ is a *k-distance paired dominating set of a graph* $G = (V, E)$ if D is k-distance dominating set of G and the induced subgraph $\langle D \rangle$ has a perfect matching. The *k-distance paired domination number* $\gamma_p^k(G)$ is the cardinality of a smallest k-distance paired dominating set of G. We give an upper bound and a lower bound on the k-distance paired domination number of a non-trivial tree T in terms of the size of T and the number of leaves in T. We also characterize the extremal trees.

Keywords: paired domination number, k-distance paired domination number, trees.
AMS Subject Classification: 05C05, 05C69.

THE SPECIAL CLASS OF THE PRIMITIVE DIGRAPHS

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A digraph D is *primitive* if there exists an integer $k > 0$ such that for all ordered pairs of vertices $u, v \in V(D)$ (not necessarily distinct), there is a walk from u to v of length k. By a walk we mean a direct path with possibly repeated vertices and arcs. The least such k is called the *exponent of the digraph* D, denoted by $\exp(D)$. The symmetric digraph is primitive, if and only if it is connected and contains odd cycles. There is known in the literature, the upper bound of the exponent of the primitive symmetric digraph D is $2\,|V(D)| - 2$. Every symmetric digraph can be obtained from a graph by replacing each edge by two arcs, one in each direction. In this sense we consider the primitivity of graphs.

We define the partial join $G_0(G) + H$ of graph G and H with respect to the induced subgraph $G_0 \leqslant G$ in the following way: it is a graph with the set of vertices $V(G) \cup V(H)$ and the set of edges $E(G) \cup E(H) \cup E_0$, where $E_0 = \{uv : u \in V(G_0) \wedge v \in V(H)\}$.

We consider the graphs H, G_0 and G from special classes of graphs. Our aim is to prove that $\exp(G_0(G) + H) \in \{\exp(G), \exp(G) + 1, \exp(G) + 2\}$.

Keywords: primitive graph, exponent of the primitivity.
AMS Subject Classification: 05C45.

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ON REED'S CONJECTURE ABOUT ω, Δ AND χ

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For a given graph G, the clique number $\omega(G)$, the chromatic number $\chi(G)$ and the maximum degree $\Delta(G)$ satisfy $\omega(G) \leqslant \chi(G) \leqslant \Delta(G) + 1$. In 1941 Brooks has shown that complete graphs and odd cycles are the only graphs attaining the upper bound $\Delta(G) + 1$.

In 1998 Reed posed the following conjecture

Conjecture. *For any graph G of maximum degree Δ,*

$$\chi(G) \leqslant \lceil \frac{\Delta + 1 + \omega}{2} \rceil.$$

The Chvátal graph, the smallest 4-regular, triangle-free graph of order 12 with chromatic number 4, shows that the rounding up in this conjecture is necessary. In this talk we will present some old and new partial solutions for this conjecture.

In particular we will show that the conjecture is true

(1) for all graphs of order $n \leqslant 12$,
(2) for all graphs with $\Delta(G) = n - k$ and $\alpha(G) \geqslant k$ for fixed k and
(3) for all graphs with $n - 5 \leqslant \Delta(G) \leqslant n - 1$.

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SELF-COMPLEMENTARY k-UNIFORM HYPERGRAPHS

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A k-uniform hypergraph $H = (V; E)$ is called *self-compementary* if there is a permutation $\sigma : V \to V$, called *self-complementing*, such that for every k-subset e of V, $e \in E$ if and only if $\sigma(e) \notin E$. In other words, H is isomorphic with $H' = (V; \binom{V}{k} - E)$.
In the paper, for every k, $1 \leqslant k \leqslant n$, we give a characterisation of self-complementig permutations of k-uniform self-complementary hypergraphs of order n. This characterisation implies the well known results for self-complementing permutations of graphs, given independently in the years 1962–1963 by Sachs and Ringel, and those obtained for 3-uniform hypergraphs by Kocay, for 4-uniform hypergraphs by Szymański, and for general (not uniform) hypergraphs by Zwonek.

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CIRCULAR CHROMATIC INDEXES OF GRAPHS

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Suppose G is a graph and $r \geqslant 2$ is a real number. A (*circular*) r-*coloring* of G is a mapping $c : V(G) \to [0, r)$ such that for each edge xy of G, $1 \leqslant |c(x) - c(y)| \leqslant r - 1$. The *circular chromatic number* $\chi_c(G)$ of G is defined as

$$\chi_c(G) = \inf\{r : \text{ there is a circular } r\text{-coloring of } G\}.$$

It follows easily from the definition that $\chi_c(G) \leqslant \chi(G)$ and it is also easy to show that $\chi_c(G) > \chi(G) - 1$. This implies that $\chi(G) = \lceil \chi_c(G) \rceil$. We say $\chi_c(G)$ is a refinement of $\chi(G)$, and $\chi(G)$ is an approximation of $\chi_c(G)$. The study of circular chromatic number of graphs has attracted considerable attention, and has become an important branch of chromatic graph theory. In this talk, I will concentrate on the circular chromatic number of line graphs. Let $L(G)$ be the line graph of G. Then $\chi_c(L(G))$ is called the *circular chromatic index* of G and is denoted by $\chi'_c(G)$. Similarly, the circular chromatic index of a graph G is a refinement of its chromatic index. It follows from Vizing Theorem that $\Delta(G) \leqslant \chi'_c(G) \leqslant \Delta(G) + 1$ (assuming that G is a simple graph). The following results have been proved recently by several groups of authors:

(1) If G is cubic and 2-edge connected, then $\chi'_c(G) \leqslant 11/3$.

(2) If G is cubic and has large girth, then $\chi'_c(G) < 3 + \epsilon$.

(3) If G has maximum degree k and has large girth, then $\chi'_c(G) < k + \epsilon$. Here $\epsilon > 0$ approaches 0 as the girth goes to infinity.

In this talk I shall sketch a proof of the result (3).

ON SOME ARBITRARILY VERTEX DECOMPOSABLE GRAPHS

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A graph G of order n is called *arbitrarily vertex decomposable* if for each sequence $(n_1, ..., n_k)$ of positive integers such that $n_1 + ... + n_k = n$ there exists a partition $(V_1, ..., V_k)$ of the vertex set of G such that for each $i \in \{1, ..., k\}$ V_i induces a connected subgraph of G on n_i vertices. Arbitrarily vertex decomposable graphs have been considered in some papers ([1], [2], [3]). The problem of deciding whether a given graph is arbitrarily vertex decomposable is NP-complete [4]. However, each traceable graph is arbitrarily vertex decomposable. It is also clear that a connected graph is arbitrarily vertex decomposable if its spanning tree is arbitrarily vertex decomposable. In the report some families of arbitrarily vertex decomposable trees and unicyclic graphs will be characterised.

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