10th WORKSHOP 0N GRAPH THEORY

# COLOURINGS, INDEPENDENCE AND DOMINATION

# **CID**

Karpacz 2003, September 22-26

# ABSTRACTS

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## AN ALGORITHM FOR GENERATING REGULAR GRAPHS

Krystyna T. Balińska, Tomasz Lubiński, Krzysztof T. Zwierzyński
*Technical University of Poznań, Poland*

Several methods of generating labeled regular graphs uniformly are known (e.g. [1]-[4]). In [5] the problem of obtaining regular graphs as an outcome of the random process for graphs with bounded degree is discussed.

A randomized algorithm for generating regular graphs, proposed in [6], will be described. Its main idea is as follows. Given $n$ - the number of vertices and $r$ - the degree of regularity, initially colour all vertices blue. Next, for each vertex $i$, $1 \leq i \leq n$, uniformly select neighbours of $i$ from $L_i$, the set of available vertices relative to $i$. If the degree of a vertex $i$ reaches $r$, then colour this vertex red. If $\deg(i) < r$ and $L_i$ is empty, then for the two cases: (a) $\deg(i) < r-1$, and (b) $\deg(i) = r-1$ a procedure of exchanging edges is applied. The algorithm terminates when all vertices are coloured red. The computational complexity and other properties of this algorithm will be given.

**Keywords:** regular graphs.
**AMS Subject Classification:** 05C30.

### References

[1] N.C. Wormald, *Generating random regular graphs*, J. Algorithms (1984) 247–280.

[2] M. Jerrum, A. Sinclair, *Fast uniform generation of regular graphs*, Theoretical Computer Science **73** (1990) 91–100.

[3] B.D. McKay, N.C. Wormald, *Uniform generation of random regular graphs of moderate degree*, J. Algorithms **11** (1990) 325–338.

[4] A. Steger, N.C. Wormald, *Generating random regular graphs quickly*, Combinatorics, Probab. and Comput. **8** (1999) 377–396.

[5] K.T. Balińska, *Algorithms for random graphs with bounded degree*, The Technical University of Poznań Press **314**, Poznań (1996).

[6] T. Lubiński, *The algorithm genregTL*, in: K.T. Balińska, K.T. Zwierzyński, Projektowanie algorytmów grafowych, The Technical University of Poznań Press, Poznań (2002) 60–61 (in Polish).

## RAMSEY NUMBERS FOR SOME DISJOINT CYCLES

Halina Bielak

*Maria Sklodowska-Curie University, Lublin, Poland*

Let $F, G, H$ be simple graphs with at least two vertices. The *Ramsey number* $R(G, H)$ is the smallest integer $n$ such that in arbitrary two-colouring (say red and blue) of $K_n$ a red copy of $G$ or a blue copy of $H$ is contained (as subgraphs). If $G \cong H$ we write $R(G)$ instead of $R(G, G)$.

We study the Ramsey number $R(G)$, where $G$ is a disjoint union of some cycles.

**Keywords:** cycle, Ramsey number, union of cycles.
**AMS Subject Classification:** 05C.

# GENERATING LABELED CUBIC PLANAR GRAPHS UNIFORMLY AT RANDOM

Manuel Bodirsky[1], Clemens Gröpl[2] and Mihyun Kang[1]
[1]Humboldt University Berlin, Germany
[2]Free University Berlin, Germany

McKay and Wormald [1] showed how to generate random regular graph of moderate degree uniformly at random in an expected polynomial time. But little is known if we restrict our attention to random regular planar graphs: a planar graph is a graph which can be embedded in the plane, whereas a planar map is an embedded graph.

We present an expected polynomial time algorithm to generate labeled 3-regular (i.e. cubic) planar graphs uniformly at random. We derive recurrence formulas that exactly count all such graphs on $n$ vertices, based on a decomposition along the connectivity structure of the graph into 1-, 2-, 3-connected components. The recurrence formulas can be evaluated in polynomial time using dynamic programming and they immediately yield the generation procedures. As a final step of enumeration and generation, we make use of the fact that a 3-connected cubic planar graph has a unique embedding and the dual of a 3-connected cubic planar map is a 3-connected planar triangulation. We thus employ the number of 3-connected planar triangulations by Tutte [3] and the uniform generation algorithm of 3-connected planar triangulations by Schaeffer [2].

**Keywords:** cubic planar graphs, uniform generation, exact enumeration, decomposition, connectivity.
**AMS Subject Classification:** 05C30, 68R10.

## References

[1] B.D. McKay, N.C. Wormald, *Uniform generation of random regular graphs of moderate degree*, J. of Algorithms **11** (1990) 325–338.

[2] G. Schaeffer, *Random sampling of large planar maps and convex polyhedra*, in: Proc. of the thirty-first annual ACM symposium on theory of computing (STOC'99) (1999) 760–769.

[3] W.T. Tutte, *A census of planar triangulations*, Canad. J. Mathematics **14** (1962) 21–38.

# GAME PARTITIONS OF GRAPHS

Mieczysław Borowiecki and Elżbieta Sidorowicz
*University of Zielona Góra, Poland*

We denote by $\mathcal{I}$ the class of all finite simple graphs. A *graph property* is a nonempty isomorphism-closed subclass of $\mathcal{I}$. A property $\mathcal{P}$ is called (*induced*) *hereditary* if it is closed under (induced) subgraphs.

Given hereditary properties $\mathcal{P}_1, \mathcal{P}_2, ..., \mathcal{P}_n$, a *vertex* $(\mathcal{P}_1, \mathcal{P}_2, ..., \mathcal{P}_n)$*-partition* (*generalized colouring*) of a graph $G \in \mathcal{I}$ is a partition $(V_1, V_2, ..., V_2)$ of $V(G)$ such that for $i = 1, 2, ..., n$ the induced subgraph $G[V_i]$ has the property $\mathcal{P}_i$.

We consider the version of colouring game. We combine: the colouring game and generalised colouring of graphs as follows. The two players are Alice and Bob and they play alternatively with Alice having the first move. Given a graph $G$ and an ordered set of hereditary properties $(\mathcal{P}_1, \mathcal{P}_2, ..., \mathcal{P}_n)$. The players take turns colouring of $G$ with colours from $\{1, 2, ..., n\}$ such that for each $i = 1, 2, ..., n$ the induced subgraph $G[V_i]$ ($V_i$ is the set of vertices of $G$ with colour $i$) has property $\mathcal{P}_i$ after each move of players. If after $|V(G)|$ moves the graph $G$ is $(\mathcal{P}_1, \mathcal{P}_2, ..., \mathcal{P}_n)$-partitioned (generalized coloured) then Alice wins. Above defined game we will call $(\mathcal{P}_1, \mathcal{P}_2, ..., \mathcal{P}_n)$*-game*.

In this talk some new results and open problems on the $(\mathcal{P}_1, \mathcal{P}_2, ..., \mathcal{P}_n)$-game will be presented.

**Keywords:** game generalized colouring, hereditary property.
**AMS Subject Classification:** 05C15, 05C75.

## References

[1] H.L. Bodlander *On the complexity of some colorings games*, Internat. J. Found. Comput. Sci. **2** (1991) 133–147.

[2] M. Borowiecki, P. Mihók, *Hereditary properties of graphs*, in: V.R. Kulli, ed., Advances in Graph Theory (Vishwa International Publication, Gulbarga, 1991) 41–68.

[3] M. Borowiecki, I. Broere, M. Frick, P. Mihók, G. Semanišin, *A survey of hereditary properties of graphs*, Discussiones Mathematicae Graph Theory **17** (1997) 5–50.

## GAME LIST COLOURING OF GRAPHS

Mieczysław Borowiecki and Elżbieta Sidorowicz
*University of Zielona Góra, Poland*

Let $N$ denote the set of positive integers and $2^N$ denote the power set of $N$. A *list assignment* $L$ for $G$ is a function $L : V \to 2^N$. A function $f : V \to N$ is an *L-colouring* of $G$ if $f(v) \neq f(u)$ whenever $vu \in E(G)$ and $f(v) \in L(v)$ for all $v \in V(G)$. If $G$ admits an $L$-colouring, then $G$ is said to be *L-colourable*. If $|L(v)| = k$ for every $v \in V(G)$ then $L$ is a *k-list-assignment*.

We consider the two-players game defined as follows. Let $(G, L)$ be a graph $G$ with a list assignment $L$. The two players are Alice and Bob and they play alternatively with Alice having the first move. Alice's goal is to provide an $L$-colouring of $G$ and Bob's goal is to prevent her from doing so. A move consists in choosing an uncoloured vertex $v$ and assigning it a colour from the set $L(v)$. This game will be called *game list colouring*. We say that $(G, L)$ is *game list colourable* if Alice has a winning strategy. The *game choice number* of $G$, denoted by $\mathrm{ch}_g(G)$, is defined as the least $k$ such that Alice has a winning strategy for any $k$-list-assignment of $G$.

We characterize the class of graphs with $\mathrm{ch}_g(G) \leq 2$ and determine the game choice number for some class of graphs.

**Keywords:** game list colouring, game choice number.
**AMS Subject Classification:** 05C15, 05C75.

### References

[1] H.L. Bodlander *On the complexity of some colorings games*, Internat. J. Found. Comput. Sci. **2** (1991) 133–147.

[2] P. Erdós, A.L. Rubin and H. Taylor, *Choosability in graphs*, in: Proc. West Coast Conf. on Combin., Graph Theory and Computing, Congressus Numerantium XXVI (1979) 125–157.

## SQUARE-FREE COLORINGS OF GRAPHS

Boštjan Brešar and Sandi Klavžar
*University of Maribor, Slovenia*

Let $G$ be a graph and $c$ a coloring of its edges. If the sequence of colors along a walk of $G$ is of the form $a_1, \ldots, a_n, a_1, \ldots, a_n$, the walk is called a *square walk*. We say that the coloring $c$ is *square-free* if any open walk is not a square and call the minimum number of colors needed so that $G$ has a square-free coloring a *walk Thue number* and denote it by $\pi_w(G)$. This concept is a variation of the Thue number introduced in [1].

Using the walk Thue number several results of [1] are extended. The Thue number of some complete graphs is extended to Hamming graphs. This result (for the case of hypercubes) is used to show that if a graph $G$ on $n$ vertices and $m$ edges is the subdivision graph of some graph, then $\pi_w(G) \leq n - \frac{m}{2}$. Graph products are also considered. An inequality for the Thue number of the Cartesian product of trees is extended to arbitrary graphs and upper bounds for the (walk) Thue number of the direct and the strong products are also given. Using the latter results the (walk) Thue number of complete multipartite graphs is bounded which in turn gives a bound for arbitrary graphs in general and for perfect graphs in particular.

**Keywords:** coloring, non-repetitive, Thue number, walk.
**AMS Subject Classification:** 05C15, 11B75.

### References

[1] N. Alon, J. Grytczuk, M. Hałuszczak, O. Riordan, *Non-repetitive colorings of graphs*, Random Structures Algorithms **21** (2002) 336–346.

## PARTITION PROBLEMS OF PLANAR GRAPHS

Izak Broere and Bonita S. Wilson

*Rand Afrikaans University, Johannesburg, Republic of South Africa*

We follow the notation and terminology of M. Borowiecki et al. in [1]. A *graph property* is a non-empty isomorphism-closed subset of the set of all mutually non-isomorphic graphs $\mathcal{I}$. A property $\mathcal{P}$ of graphs is called *additive* if it is closed under the union of graphs, i.e., if every connected component of a graph $G$ has property $\mathcal{P}$, then $G \in \mathcal{P}$. The property $\mathcal{P}$ is called *hereditary* if it is closed under taking subgraphs, i.e., if $H \subseteq G$ and $G \in \mathcal{P}$ then $H \in \mathcal{P}$ too.
We note that, for each positive integer $k$, $\mathcal{T}_k = \{G \in \mathcal{I} \mid G$ contains no subgraph homeomorphic to $K_{k+2}$ or $K_{\lfloor \frac{k+3}{2} \rfloor, \lceil \frac{k+3}{2} \rceil}\}$ is an additive hereditary graph property. Furthermore, $\mathcal{T}_3$ is, by Kuratowski's Theorem (see [2]), the set of planar graphs.
For properties $\mathcal{P}_1$ and $\mathcal{P}_2$ a *vertex* $(\mathcal{P}_1, \mathcal{P}_2)$*-partition* of a graph $G$ is a partition $V_1, V_2$ of $V(G)$ such that for each $i$ the induced subgraph $G[V_i]$ has property $\mathcal{P}_i$. (The empty set will be regarded as a set inducing a subgraph with any property.)
For given properties $\mathcal{P}_1$ and $\mathcal{P}_2$ we define the *product* by $\mathcal{P}_1 \circ \mathcal{P}_2 = \{G \in \mathcal{I} \mid G$ has a vertex $(\mathcal{P}_1, \mathcal{P}_2)$-partition$\}$.
In this paper we present some results of the form $\mathcal{T}_3 \not\subseteq \mathcal{P} \circ \mathcal{Q}$ by showing the existence of suitable planar graphs which do not admit the required $(\mathcal{P}, \mathcal{Q})$-partition for some well-known properties $\mathcal{P}$ and $\mathcal{Q}$. We also present results of the form: If $\mathcal{T}_3 \subseteq \mathcal{P} \circ \mathcal{Q}$ with $\mathcal{Q}$ fixed, then $\mathcal{P}$ has to satisfy certain requirements.

**Keywords:** planar graph, hereditary property.
**AMS Subject Classification:** 05C10, 05C15.

### References

[1] M. Borowiecki, I. Broere, M. Frick, P. Mihók, G. Semanišin, *A survey of hereditary properties of graphs*, Discussiones Mathematicae Graph Theory **17** (1997) 2–38.

[2] K. Kuratowski, *Sur le problème des courbes gauches en topologie*, Fund. Math. **15** (1930) 271–283.

## ARITHMETICALLY MAXIMAL INDEPENDENT SETS IN INFINITE GRAPHS

Stanisław Bylka

*Polish Academy of Sciences, Warsaw, Poland*

Let $\Gamma(G)$ be the set of all independent vertices of a graph $G$. An *arithmetically maximal independent set* (a.m.i.s.) of $G$ is a set $S \in \Gamma(G)$ such that for every pair of finite sets of vertices $A \subset S$ and $B \cap S = \emptyset$, if $(S \setminus A) \cup B \in \Gamma(G)$ then $|A| \geq |B|$. For any graph $G$, a *König covering* of $G$ is an ordered pair $(S, \mathcal{K})$ such that $S \in \Gamma(G)$, $\mathcal{K}$ is a cliques covering of vertices of $G$, and $S$ consists a vertex from every clique of $\mathcal{K}$. Clearly, if $(S, \mathcal{K})$ is a König covering of $G$ then $|S| = |\mathcal{K}|$. We know that

(1) if $(S, \mathcal{K})$ is a König covering of $G$ then $S$ is an a.m.i.s. of $G$,

(2) every finite graph $G$ has an a.m.i.s. of vertices.

Every bipartite graph has an a.m.i.s. because the König duality theorem and its extensions for countable and uncountable bipartite graphs. This paper is devoted to the problem of a.m.i.s. of vertices in graphs. A negative answer is given for tripartite graphs. A positive solution is given for line graphs and graphs having locally finite clique covering. Some counter-examples are also presented.

**Keywords:** countale graphs, bipartite graphs, line graphs, independent sets, coverings.
**AMS Subject Classification:** 05D15.

# ADDITIVE HEREDITARY PROPERTIES OF HYPERGRAPHS BASED ON CHROMATIC SUMS OF HYPERGRAPHS

Ewa Drgas-Burchardt and Anna Fiedorowicz
*University of Zielona Góra, Poland*

The *chromatic sum of a hypergraph H* is the smallest sum of colors among all proper colorings using natural numbers. This notion for graphs was introduced by E. Kubicka in [2]. We construct additive hereditary properties of hypergraphs based on chromatic sums of hypergraphs. Namely, a hypergraph $H$ has a property $\Sigma_k^a$ if and only if every component of $H$ has a chromatic sum bounded above by $\binom{k+2}{2}$. We analize the properties $\Sigma_k^a$, $k = 1, 2, \ldots$ in terms of maximal hypergraphs and minimal forbidden subhypergraphs. Besides, we give the generating function for a sequence describing a number of maximal graphs of properties $\Sigma_k^a$, $k = 1, 2, \ldots$

**Keywords:** hypergraph, chromatic sum, additive hereditary property.
**AMS Subject Classification:** 05C65, 05C15.

## References

[1] M. Borowiecki, P. Mihók, *Hereditary properties of graphs*, Advances in Graph Theory, ed. V.R. Kulli, Vishwa International Publications, Gulbarga (1991) 41–68.

[2] E. Kubicka, A.J. Schwenk, *An introduction to chromatic sums*, Proceedings of the Seventeenth, Annual ACM Computer Sciences Conference, ACM Press (1989) 39–45.

[3] J. Mitchem, P. Morriss, *On the cost chromatic number of graphs*, Discrete Mathematics **171** (1997) 201–211.

## ON PANCHROMATIC COLORINGS OF HYPERGRAPHS

Ewa Drgas-Burchardt
*University of Zielona Góra, Poland*

and

Ewa Łazuka
*Technical University of Lublin, Poland*

Let $H$ be a hypergraph and $k \geq 2$ be a positive integer. A vertex $k$-coloring of $H$ is *panchromatic* if each of the $k$ colors is used on every egde of $H$ [1,2]. The number of panchromatic $k$-colorings of $H$ is given by a polynomial $f_k(H, \lambda)$ of degree $|V(H)|$ in $\lambda$, called *the $k$-panchromatic polynomial* of $H$.

We present the method of calculating the $k$-panchromatic polynomial of any hypergraph. It uses the partitions of a graph induced by a $k$-subset of a fixed edge of $H$ into stable sets. We apply this method to several types of hypergraphs. We also study some coefficients of $f_k(H, \lambda)$.

**Keywords:** panchromatic coloring of a hypergraph, chromatic polynomial of a hypergraph.
**AMS Subject Classification:** 05C15.

### References

[1] A.V. Kostochka, *On a theorem of Erdös, Rubin, and Taylor on choosability of complete bipartite graphs*, The Electronic Journal of Combinatorics **9** (2002) #N9.

[2] A.V. Kostochka, D.R. Woodall, *Density conditions for panchromatic colourings of hypergraphs*, Combinatorica **21** (2001) 515–541.

## HAMILTONIAN PATH SATURATED GRAPHS WITH MINIMUM SIZE

Aneta Dudek[1], Gyula Y. Katona Jr.[2] and A. Paweł Wojda[1]
[1] *AGH University of Science and Technology, Kraków, Poland*
[2] *Budapest University of Technology, Hungary*

A graph $G$ said to be *hamiltonian path saturated* (HPS for short), if $G$ has no hamilltonian path but any addition of a new edge in $G$ creates in $G$ hamiltonian path.

In 1977 Bondy proved that an HPS graph of order $n$ has the size at most $\binom{n-1}{2}$ and, for $n \geq 6$, the only HPS graph of order $n$ and size $\binom{n-1}{2}$ is $K_{n-1} \cup K_1$. Denote by $sat(n, HP)$ the minimum size of an HPS graph of order $n$. We prove that $sat(n, HP) \geq \lfloor \frac{3n-1}{2} \rfloor - 2$. Using the some properties of Isaacs' snarks we give, for every $n \geq 52$, an HPS graph $G_n$ of order $n$ and size $\lfloor \frac{3n-1}{2} \rfloor$. This proves $sat(n, HP) \leq \lfloor \frac{3n-1}{2} \rfloor$. We consider also the $m$-path cover saturated graphs and the $P_m$-saturated graphs with small size.

## NEW VALUES AND BOUNDS FOR MULTICOLOR RAMSEY NUMBERS

Tomasz Dzido
*University of Gdańsk, Poland*

The *Ramsey number* $R(G_1, G_2, G_3)$ is the smallest number $n$ such that in every 3-coloring of the edges of complete graph $K_n$ with color red, blue and green a red subgraph $G_1$, a blue subgraph $G_2$ or a green subgraph $G_3$ occurs. In this note we consider Ramsey numbers $R(P_3, C_k, C_m)$, where $P_3$ is the path on 3 vertices, and $C_i$ is the cycle on i vertices. In addition, we present new bounds for $R(C_k, C_k, C_k)$ where $k$ is even positive number. In this paper we will present new results in this field as well as some conjectures.

**Keywords:** Ramsey numbers.

## THE UNIQUELY ONE-ONE REALIZABLE DEGREE SETS BY MINIMUM IRREGULAR DIGRAPHS

Zyta Dziechcińska-Halamoda, Zofia Majcher, Jerzy Michael
*University of Opole, Poland*

and
Zdzisław Skupień
*AGH University of Science and Technology, Kraków, Poland*

A $k$-digraph (with multiplicities of arcs at most $k$, loops being allowed) is called *irregular* if different vertices have distinct degree pairs. By a digraph we mean a 1-digraph without loops. An oriented graph is a digraph without 2-cycles. A minimum irregular digraph is an irregular digraph with the minimum size. In paper [1] sets of degree pairs of minimum irregular digraphs are characterized. In this talk we give the list of all sets which have the unique one-one realization in the classes of 1-digraphs, digraphs, oriented 1-graphs and oriented graphs.

**Keywords:** irregular digraph, degree pair, unique realization.
**AMS Subject Classification:** 05C.

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# EQUITABLE COLORING OF GRAPH PRODUCTS

Hanna Furmańczyk
*University of Gdańsk, Poland*

A graph is *equitably k-colorable* if its vertices can be partitioned into $k$ independent sets in such way that the number of vertices in any two sets differ by at most one. The smallest $k$ for which such coloring exists is known as the *equitable chromatic number* of $G$ and denoted by $\chi_=(G)$. It is interesting to note, that if a graph $G$ is equitably $k$-colorable, it does not imply that it is equitably $(k+1)$-colorable. The smallest integer $k$ for which $G$ is equitably $k'$-colorable for all $k' \geq k$ is called the *equitable chromatic threshold* of $G$ and denoted by $\chi_=^*(G)$. In the paper we establish the equitable chromatic number and the equitable chromatic threshold for some products of some particular graphs. We extend the results from [2] for Cartesian, weak and strong tensor products, denoted by $G_1 \times G_2$, $G_1 \otimes G_2$, $G_1 \Box G_2$, respectively [1].

**Keywords:** equitable coloring, graph products.
**AMS Subject Classification:** 05C15, 68R10.

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## INDUCED RAMSEY CLASSES

Izolda Gorgol
*Technical University of Lublin, Poland*

and
Mariusz Hałuszczak
*University of Zielona Góra, Poland*

Let $F$ be a graph and $G$ be its subgraph. We colour the edges of the graph $F$ with two colours and look for a monochromatic induced subgraph $G'$ isomorphic to $G$. We consider two types of appearance of $G$ as an induced subgraph. We say that $G'$ is a *strong* copy of $G$ iff $G'$ is an induced subgraph of $F$, monochromatic and isomophic to $G$. We say that $G'$ is a *weak* copy of $G$ iff $G'$ is isomorphic to $G$ and induced in monochromatic subgraph of $F$. Let $\mathcal{G}$ and $\mathcal{H}$ be the families of graphs and $F$ be a graph.

The symbol $F \xrightarrow{i_s} (\mathcal{G},\mathcal{H})$ means that any 2-colouring (say red and blue) of the edges of the graph $F$ leads to either a strong red copy of a certain graph from the family $\mathcal{G}$ or a blue strong copy of a certain graph from family $\mathcal{H}$. Similarly the symbol $F \xrightarrow{i_w} (\mathcal{G},\mathcal{H})$ means that any 2-colouring of the edges of the graph $F$ leads to either a weak red copy of a certain graph from the family $\mathcal{G}$ or a blue weak copy of a certain graph from family $\mathcal{H}$.

We call a graph $F$ *strong critical* for $(\mathcal{G},\mathcal{H})$ iff $F \xrightarrow{i_s} (\mathcal{G},\mathcal{H})$ and for each proper induced subgraph $F' \preceq F$ holds $F' \overset{i_s}{\not\longrightarrow} (\mathcal{G},\mathcal{H})$. The class of all strong critical graphs for $(\mathcal{G},\mathcal{H})$ we denote by $\Im\Re_s^c(\mathcal{G},\mathcal{H})$.

We call a graph $F$ *strong global minimal* for $(\mathcal{G},\mathcal{H})$ iff $F \xrightarrow{i_s} (\mathcal{G},\mathcal{H})$ and for each proper subgraph $F' \subseteq F$ holds $F' \overset{i_s}{\not\longrightarrow} (\mathcal{G},\mathcal{H})$. The class of all strong global minimal graphs for $(\mathcal{G},\mathcal{H})$ we denote by $\Im\Re_s^g(\mathcal{G},\mathcal{H})$. We call a graph $F$ *strong local minimal* for $(\mathcal{G},\mathcal{H})$ iff $F \xrightarrow{i_s} (\mathcal{G},\mathcal{H})$ and for each $e \in E(F)$ holds $F - e \overset{i_s}{\not\longrightarrow} (\mathcal{G},\mathcal{H})$. The class of all strong local minimal graphs for $(\mathcal{G},\mathcal{H})$ we denote by $\Im\Re_s^l(\mathcal{G},\mathcal{H})$.

Analogously we define classes of all weak critical $\Im\Re_w^c(\mathcal{G},\mathcal{H})$, weak global minimal $\Im\Re_w^g(\mathcal{G},\mathcal{H})$ and weak local minimal $\Im\Re_w^l(\mathcal{G},\mathcal{H})$ graphs for $(\mathcal{G},\mathcal{H})$.

In the talk we show some properties of graphs in certain induced classes.

**Keywords:** Ramsey class, induced subgraph.
**AMS Subject Classification:** 05D10, 05C55.

## ON DOMINATION IN GRAPHS

Frank Göring

*Technical University of Chemnitz, Germany*

and

Jochen Harant

*Technical University of Ilmenau, Germany*

For a finite undirected graph $G$ on $n$ vertices continuous optimization problems taken over the $n$-dimensional cube are presented and it is proved that their optimum values equal the domination number $\gamma$ of $G$. An efficient approximation method is developed and known upper bounds on $\gamma$ are slightly improved.

**Keywords:** graph, domination.
**AMS Subject Classification:** 05C35.

## SIERPIŃSKI GRAPHS: $L(2,1)$-COLORINGS AND PERFECT CODES

Sylvain Gravier, Sandi Klavžar* and Michel Mollard
*University of Maribor, Slovenia

Sierpiński graphs $S(n,k)$ generalize the Tower of Hanoi graphs - the graph $S(n,3)$ is isomorphic to the graph of the Tower of Hanoi with $n$ disks. These graphs were introduced in [1] and further studied in [2].

The $\lambda$-number of a graph $G$ is the minimum value $\lambda$ such that $G$ admits a coloring/labeling with colors from $\{0,1,\ldots,\lambda\}$ where vertices at distance two get different colors and adjacent vertices get colors that are at least two apart. The main result of this talk asserts that for any $n \geq 2$ and any $k \geq 3$, $\lambda(S(n,k)) = 2k$. To obtain the result (1-perfect) codes (1-perfect codes are also known as efficient dominating sets) in Sierpiński graphs were studied in detail. In particular a new proof of their (essential) uniqueness will be mentioned.

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# COLOURING OF CONFIGURATIONS

Harald Gropp
*Universität Heidelberg, Germany*

A *configuration* is a linear regular uniform hypergraph. Since configurations are much older than hypergraphs (they are even older than graphs), usually a geometric language of points and lines is used.

The points are coloured by assigning a number (the colour) from 1 to $n$ to each point. A colouring is allowed if certain conditions are fulfilled. The usual condition is that every line (or hyperedge) contains two points with different colours. This leads to the definition of the chromatic number. In particular, the existence problem of blocking sets of configurations is related to the usual colouring.

The colouring of mixed hypergraphs (introduced by Voloshin) leads to the definition of the upper chromatic number. Here also anti-edges or C-edges are coloured such that every such C-edge contains two vertices with the same colour.

**Keywords:** configurations, hypergraphs, colouring, chromatic number, upper chromatic number.
**AMS Subject Classification:** 05B30, 05C15.

## HAPPY COLORINGS OF HYPERGRAPH COUPLES

Jarosław Grytczuk
*University of Zielona Góra, Poland*

Suppose $H = (X, \mathcal{E})$ is a hypergraph on the set of vertices $X$ and $M = (Y, \mathcal{F})$ is another hypergraph defined in some way on the set of hyperedges $\mathcal{E}$ of $H$, that is $Y = \mathcal{E}$. Then any coloring $f$ of the vertices of $H$ by colors from a set $C$ *induces* a vertex coloring of $M$ by *multisets* of colors from $C$. If the later coloring is proper we call $f$ a *happy coloring* of $H$ with respect to $M$ and denote the minimum number of colors needed by $\pi(H, M)$.

There are many situations in which such *couples* of hypergraphs appear naturally. For instance, let $X$ be the set of positive integers and let $\mathcal{E}$ be the family of all finite segments of consecutive numbers. Consider a graph $M$ in which two segments form an edge if they are *adjacent*, that is, if they are disjoint, but their union is again a segment. An old problem of Erdős asked if there is an infinite sequence over 4 symbols in which no two adjacent segments are permutations of each other. This is equivalent to decide if $\pi(H, M) = 4$ holds for a couple $(H, M)$ defined above.

Another nice example starts with a graph $G$ without isolated edges. Let $X = E(G)$ and let $\mathcal{E}$ be the family of all maximal *stars* in $G$. Then $M$ is a graph in which two stars form an edge if their centers are adjacent vertices of $G$. In a recent paper Karoński, Łuczak and Thomason proved that $\pi(H, M) \leq 183$ for any such couple.

We will present some further results and several open problems of the above type.

## GRAPHS WITH LARGE DOUBLE DOMINATION NUMBER

Michael A. Henning

*University of Natal, Pietermaritzburg, Republic of South Africa*

In a graph $G = (V, E)$, a vertex dominates itself and its neighbors. A subset of vertices $S$ of $V$ is a *double dominating set* if every vertex in $V$ is dominated at least twice. The minimum cardinality of a double dominating set of $G$ is the *double domination number* of $G$, denoted $\gamma_{\times 2}(G)$. If $G \neq C_5$ is a connected graph of order $n$ with minimum degree at least 2, then we show that $\gamma_{\times 2}(G) \leq 3n/4$ and we characterize those graphs achieving equality.

## ON COMPLETE TRIPARTITE GRAPHS ARBITRARILY DECOMPOSABLE INTO CLOSED TRAILS

Mirko Horňák and Zuzana Kocková
*P.J. Šafárik University, Košice, Slovakia*

Let $G$ be an *even* graph (all its vertices are of even degrees), let $\mathrm{Lct}(G)$ be the set of all lengths of closed trails in $G$ and let $\mathrm{Sct}(G)$ be the set of all finite sequences whose terms belong to $\mathrm{Lct}(G)$ and sum up to $|E(G)|$. The graph $G$ is said to be *arbitrarily decomposable into closed trails* (ADCT for short) if for any sequence $(l_1, \ldots, l_q) \in \mathrm{Sct}(G)$ there is a sequence $(T_1, \ldots, T_q)$ of closed trails in $G$ such that $T_i$ is of lenth $l_i$ for any $i \in \{1, \ldots, q\}$ and $\{E(T_i) : i = 1, \ldots, q\}$ is a decomposition of $E(G)$. The following graphs are known to be ADCT: $K_n$ for $n$ odd, $K_n - M_n$ for $n$ even, where $M_n$ is a perfect matching in $M_n$ (see Balister [1]), $K_{m,n}$ for $m, n$ even (see Horňák and Woźniak [2]).

**Theorem 1.** *If the graph $K_{p,q,r}$ with $p \le q \le r$ is* ADCT*, then either $(p, q, r) \in \{(1, 1, 3), (1, 1, 5)\}$ or $p = q = r$.*

**Theorem 2.** *The following graphs are* ADCT*: $K_{1,1,3}$, $K_{1,1,5}$ and $K_{n,n,n}$ with $n \in \{1, 2, 3, 4\}$ or $n = 5 \cdot 2^k$ where $k$ is a nonnegative integer.*

**Keywords:** complete tripartite graph, closed trail, edge decomposition.
**AMS Subject Classification:** 05C70.

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## ESTIMATION OF CUT-VERTICES IN EDGE-COLOURED COMPLETE GRAPHS

Adam Idzik

*Świętokrzyska Academy, Kielce and Polish Academy of Sciences, Warsaw, Poland*

Given a $k$-edge-coloured graph $G = (V, E^1, ..., E^k)$, we define $F^i = E \setminus E^i$, $G^i = (V, E^i)$, $\bar{G}^i = (V, F^i)$, where $E = \bigcup_{i \in \{1,...,k\}} E^i$ and $i \in \{1, ..., k\}$. Here $G^i$ is a monochromatic subgraph of $G$ and $\bar{G}^i$ is its complement in $G$.

The following theorem [1] is under discussion.

*Let $(E^1, ..., E^k)$ be a $k$-edge-colouring of $K_m$ ($k \geq 2$, $m \geq 4$), such that all the graphs $\bar{G}^1, \cdots, \bar{G}^k$ are connected.*

(i) *If one of the subgraphs $G^1, \cdots, G^k$ is 2-connected, say $G^i$, then $c(\bar{G}^i) \leq m - 2$ and $c(\bar{G}^j) = 0$ for $j \neq i$ $(i, j \in \{1, ..., k\})$.*

(ii) *If none of the graphs $G^1, \cdots, G^k$ is 2-connected, and one of them is connected, say $G^i$, then $c(\bar{G}^i) \leq 2$ ($i \in \{1, ..., k\}$).*

(iii) *If none of the graphs $G^1, ..., G^k$ is 2-connected, and one of them is disconnected, say $G^i$, then $c(\bar{G}^i) \leq 1$ ($i \in \{1, ..., k\}$).*

**Keywords:** complete graph, connected graph, cut-vertex, edge-colouring.
**AMS Subject Classification:** 05C35, 05C40, 68R10.

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# COMBINATORIAL LEMMAS FOR NONORIENTED PSEUDOMANIFOLDS

Adam Idzik

*Świętokrzyska Academy, Kielce and Polish Academy of Sciences, Warsaw, Poland*

and

Konstanty Junosza-Szaniawski

*Warsaw University of Technology, Poland*

For a set $A \subset R^n$ let $co\, A$ denote a convex hull of $A$. Let $P \subset R^n$ be a polytope, $Tr$ a triangulation of the polytope $P$, $V(\delta)$ the set of vertices of a simplex $\delta \in Tr$ and $V = \bigcup_{\delta \in Tr} V(\delta)$. A function $l : V \to R^n$ is called a *labeling*. A simplex $\delta \in Tr$ is *balanced* if $0 \in co\, l(V(\delta))$. We formulate general boundary conditions for the labeling $l$ to assure the existence of a balanced simplex $\delta \in Tr$. Furthermore we prove a Knaster-Kuratowski-Mazurkiewicz type theorem for polytopes and we generalize some theorems of van der Laan, Talman and Yang [1] and some theorems of Ichiishi and Idzik [2].

**Keywords:** pseudomanifold, labeling, KKM theorem.
**AMS Subject Classification:** 05B30, 47H10, 52A20, 54H25.

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## DISTANCE COLORING OF THE HEXAGONAL LATTICE

Peter Jacko and Stanislav Jendrol'
*P.J. Šafárik University, Košice, Slovakia*

Motivated by the frequency assignment problem we study the $d$-distant coloring of the vertices of an infinite plane hexagonal lattice $H$. Let $d$ be a positive integer. A *$d$-distant coloring* of a the lattice $H$ is a coloring of the vertices of $H$ such that each pair of vertices distance at most $d$ apart have different colors. The *$d$-distant chromatic number* of $H$, denoted $\chi_d(H)$, is the minimum number of colors needed for a $d$-distant coloring of $H$. We give the exact value of $\chi_d(H)$ for any $d$.

## COMPETITIVE GRAPH COLORING

Hal Kierstead
*Arizona State University, Tempe, USA*

We shall survey recent results on various versions of a graph coloring game originally introduced by Bodlander. These games are played as follows on a graph $G = (V, E)$ with a set of colors $X$. Two players, Alice and Bob, take turns playing with Alice playing first. A play consists of two parts. First the player chooses a vertex $u$ that has not yet been colored. Then the player colors $u$ with a color that is legal for $u$. Different variations of the game are obtained by changing the definition of a legal color and/or changing the number of vertices the players are allowed to color on each turn. In Bodlander's original game a color $\alpha$ is legal for an uncolored vertex $u$ if $u$ does not have any neighbors colored with $\alpha$. Alice wins the game if eventually all the vertices are legally colored; otherwise Bob wins the game when there comes a time when there is an uncolored vertex that cannot be legally colored. The *game chromatic number*, denoted by $\chi_g(G)$, of a graph $G$ is the least integer $t$ such that Alice has a winning strategy for Bodlander's original game when the game is played with $t$ colors. We shall also consider variations based on oriented coloring and relaxed coloring. For example, if $G$ is a planar graph then $\chi_g(G) \leq 17$, the oriented game chromatic number of $G$ is bounded by an absolute constant and the 132-relaxed game chromatic number of $G$ is at most 3.

**Keywords:** game chromatic number, game coloring number, planar graph.
**AMS Subject Classification:** 05C15.

## ON THE CROSSING NUMBERS OF PRODUCTS OF SMALL GRAPHS

Marián Klešč

*Technical university, Košice, Slovak Republic*

The *crossing number* $cr(G)$ of a graph $G$ is the minimum number of pairwise intersections of edges in a drawing of $G$ in the plane. Computing the crossing number of a given graph is in general an elusive problem, and the crossing numbers of very few families of graphs are known. There are known exact results on the crossing numbers of Cartesian products of paths, cycles or stars with all graphs of order four (see [1] and [2]). It thus seems natural to inquire about the crossing numbers of the products of 5-vertex graphs with cycles, paths or stars.

Let $C_n$ and $P_n$ be the cycle and the path with $n$ edges, and $S_n$ the star $K_{1,n}$. The table in [3] summarizes the known crossing numbers of Cartesian products $G_j \times P_n$, $G_j \times C_n$, and $G_j \times S_n$ for connected 5-vertex graphs $G_j$. For arbitrary large $n$, the crossing numbers of $C_n \times G_j$ are known only for 14 of 21 connected graphs $G_j$ of order five. But only for five of them the crossing numbers of $C_n \times G_j$ are known for $n = 3, 4$ and 5. The purpose of this talk is to present up to now unknown exact values of crossing numbers of the Cartesian products $C_n \times G_j$ for $n = 3, 4$ and 5. In addition, we present some methods, which have been used to prove these results.

**Keywords:** graph, cycle, drawing, crossing number.
**AMS Subject Classification:** 05C10.

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## ON $(k, l)$-KERNEL PERFECTNESS OF SPECIAL CLASSES OF DIGRAPHS

Magdalena Kucharska

*Technical University of Szczecin, Poland*

Concept on kernel-perfect digraph is well know in literature. A $(k, l)$-kernel perfect digraph is the generalization of this digraph. A directed graph $D$ such that every induced subdigraph of $D$ has a $(k, l)$-kernel is called $(k, l)$-*kernel perfect digraph*. For explanation, we recall that a subset $J \subseteq V(D)$ is a $(k, l)$-*kernel of* $D$ if

(1) for every $x, y \in J$ and $x \neq y, d_D(x, y) \geq k$ and

(2) for every $x \in V(D) \setminus J$ there exists $y \in J$ such that $d_D(x, y) \leq l$, for fixed integers $k \geq 2$, $l \geq 1$,

where $d_D(x, y)$ denotes the distance from $x$ to $y$ in $D$.

We present some necessary and sufficient conditions for special classes of digraphs to be $(k, l)$-kernel perfect digraphs.

**Keywords:** kernel, $(k, l)$-kernel, kernel perfect digraph.
**AMS Subject Classification:** 05C20.

## SPECIAL KINDS OF WELL COVEREDNESS OF GENERALIZED CARTESIAN PRODUCT OF GRAPHS

Maria Kwaśnik and Daniel Łocman
*Technical University of Szczecin, Poland*

A graph is called *well-covered* (hereinafter w-c) if all maximal independent set are the same size. The class of well-covered graphs, in which the NP-complete problem with finding a maximum independent set is trivial, it was first studied by M.D. Plummer ([3]) in 1970. There are known in the literature some subclasses of class of w-c graphs. Well-covered point critical graphs ([5]) are those $G$ whose are w-c graphs but for all vertices $v \in V(G)$, $G - v$ is not w-c graph. A w-c graph $G$ is said to be *strongly well-covered* ([2]) if for each edge $e \in E(G)$, $G - e$ is a w-c graph. A locating dominating set of a graph ([1]) is such dominating set $D$ that for every pair of vertices $u$ and $v$ not in $D$, the neighbours of $u$ in $D$ differ in at least one vertex from the neighbours of $v$ in $D$. A graph is called *well-located* ([1]) if every its independent dominating set is locating. It has been proved in [1] that well-located graphs are in fact w-c graphs. We present some results developed in this area with respect to the generalized Cartesian product of graphs $(G_1, \ldots, G_n)$ and $H$.

**Keywords:** maximum independent, well-covered, generalized cartesian product of graphs.
**AMS Subject Classification:** 05C69, 05C70.

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# EXTENDABILITY AND NEAR PERFECTNESS OF GRAPH PRODUCTS

Maria Kwaśnik and Monika Perl

*Technical University of Szczecin, Poland*

A graph $G$ is said to be $k$-*extendable* ([2]) if it is connected, has a perfect matching (a 1-factor) and every matching of cardinality $k$ in $G$ can be extend to (i.e., is a subset of) a perfect matching. We present some results which answer the question how highly extendable are two products of graphs: the corona and the Cartesian product of graphs.

A subset $S$ of vertices of a graph $G$ is called *nearly perfect* ([1]) if every vertex in $V(G)-S$ is adjacent to at most one vertex in $S$. A natural problem is to study the extremal such subsets and their cardinalities. Some results concerning these problem with respect to the corona and the Cartesian product are developed.

**Keywords:** domination, matching, products of graphs.
**AMS Subject Classification:** 05C69, 05C70.

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## DOMINATING NUMBERS IN GRAPHS WITH REMOVED EDGE OR SET OF EDGES

Magdalena Lemańska
*Gdańsk University of Technology, Poland*

It is known, that the removal of an edge from $G$ cannot decrease a domination number $\gamma(G)$ and can increase it by at most one. Thus we can write, that $\gamma(G) \leq \gamma(G-e) \leq \gamma(G)+1$ when arbitrary edge is removed. Here we present similar inequalities for weakly connected domination number $\gamma_w$ and connected domination number $\gamma_c$, i.e. we show, that $\gamma_w(G) \leq \gamma_w(G-e) \leq \gamma_w(G)+1$ and $\gamma_c(G) \leq \gamma_c(G-e) \leq \gamma_c(G)+2$ if $G$ and $G-e$ are connected.

We also show that $\gamma_w(G) \leq \gamma_w(G-E_p) \leq \gamma_w(G)+p-1$ and $\gamma_c(G) \leq \gamma_c(G-E_p) \leq \gamma_w(G)+2p-2$ if $G$ and $G-E_p$ are connected and $E_p = E(K_p)$ where $K_p \leq G$ is the complete subgraph of $G$.

The *distance* $d(u,v)$ between two vertices $u$ and $v$ in a connected graph $G$ is the length of the shortest $u-v$ path in $G$. A $u-v$ path of length $d(u,v)$ is called *$u-v$ geodesic*. A set $X \subset V$ is called *weakly convex* if for every two vertices $a, b \in X$ exists $a-b$ geodesic whose vertices also belong to $X$ and $X$ is called *convex* if for every two vertices $a, b \in X$, vertices from every $a-b$ geodesic also belong to $X$. We define two new domination parameters $\gamma_{wcon}$ and $\gamma_{con}$.

The *weakly convex domination number* of $G$, denoted $\gamma_{wcon}(G)$, is $\min\{|D| : D$ is a minimal weakly convex dominating set of $G\}$, while the *convex domination number* of $G$, denoted $\gamma_{con}(G)$, is $\min\{|D| : D$ is a minimal convex dominating set of $G\}$.

For numbers $\gamma_{wcon}$ and $\gamma_{con}$ we show, that differences $\gamma_{wcon}(G)-\gamma_{wcon}(G-e)$, $\gamma_{wcon}(G-e)-\gamma_{wcon}(G)$, $\gamma_{con}(G)-\gamma_{con}(G-e)$, $\gamma_{con}(G-e)-\gamma_{con}(G)$ can be arbitrarily large.

**Keywords:** connected dominating number, weakly connected dominating number, edge removal.
**AMS Subject Classification:** 05C05, 05C69.

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## DOMINATING BIPARTITE SUBGRAPHS IN GRAPHS

Danuta Michalak
*University of Zielona Góra, Poland*

A graph $G$ is *hereditarily dominated* by a class of connected graphs $\mathcal{D}$ if each connected induced subgraph of $G$ contains dominating induced subgraph belonging to $\mathcal{D}$. In this paper we determine graphs hereditarily dominated by a class $\mathcal{D}_1 = \{K_1, K_{1,1}, ..., K_{n-1,n} : n \geq 3\}$, $\mathcal{D}_2 = \{G : G = K_{i,j}, i \geq 0, j \geq 1\}$ and $\mathcal{D}_3 = \{G : G \text{ is a connected bipartite graph}\}$.

**Keywords:** dominating set, dominating subgraph, induced forbidden subgraph.
**AMS Subject Classification:** 05C69, 05C38.

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## CHARACTERISTIC OF HEREDITARY GRAPH PROPERTIES

Peter Mihók

*Slovak Academy of Sciences and Technical University, Košice, Slovak Republic*

Let $\preceq$ be any well-founded partial order on the class of simple graphs $\mathcal{I}$. A graph theoretical invariant $\rho$ is called $\preceq$-monotone whenever for any pair $G_1, G_2$ of graphs $G_1 \preceq G_2$ implies $\rho(G_1) \leq \rho(G_2)$.

Any $\preceq$-hereditary property $\mathcal{P}$ can be uniquely determined by the set of *minimal forbidden subgraphs* of the property $\mathcal{P}$ defined in the following way:

$$\boldsymbol{F}(\mathcal{P}) = \{G \in \mathcal{I} \setminus \mathcal{P} : \text{ each graph } H \prec G \text{ belongs to } \mathcal{P}\}.$$

Note that $\boldsymbol{F}(\mathcal{P})$ may be finite or infinite. Another possibility to determine a hereditary property $\mathcal{P}$ provide $\preceq$-generating sets of $\mathcal{P}$ defined in the following way: $\mathcal{G}$ is a generating set of $\mathcal{P}$ if and only if for any graph $G \in \mathcal{P}$ there is a graph $H \in \mathcal{G}$ such that $G \preceq H$. More details on generating sets can be found in [1, 6].

In the study of generalized colourings of graphs the partial order "to be a subgraph" is of the most importance. For this partial order we shall omit the symbol "$\subseteq$" and simply say that a property is hereditary. A property is said to be additive if it is closed under taking disjoint union of graphs.

**Example 1.** For an illustration we list some additive hereditary properties.

$$\begin{aligned}
\mathcal{O} &= \{G \in \mathcal{I} : G \text{ is edgeless, i.e. } E(G) = \emptyset\},\\
\mathcal{O}_k &= \{G \in \mathcal{I} : \text{each component of } G \text{ has at most } k+1 \text{ vertices}\},\\
\mathcal{S}_k &= \{G \in \mathcal{I} : \text{the maximum degree } \Delta(G) \leq k\},\\
\mathcal{W}_k &= \{G \in \mathcal{I} : \text{the order of the longest path } \tau(G) \leq k+1\},\\
\mathcal{D}_k &= \{G \in \mathcal{I} : G \text{ is } k\text{-degenerate, i.e. the minimum degree } \delta(H) \leq k\\
&\qquad\qquad \text{for each } H \subseteq G\},\\
\mathcal{I}_k &= \{G \in \mathcal{I} : G \text{ does not contain } K_{k+2}\},\\
\mathcal{O}^k &= \{G \in \mathcal{I} : G \text{ is } k\text{-colorable}\}.
\end{aligned}$$

Let us introduce for an arbitrary graph theoretical invariant $\rho$ and a $\preceq$-hereditary property $\mathcal{P}$ the following characteristics (see [2, 3, 4]):

$$\rho(\mathcal{P}) = \min\{\rho(F) : F \in \boldsymbol{F}(\mathcal{P})\} \quad \text{and} \quad c_\rho(\mathcal{P}) = \sup\{\rho(F) : F \in \mathcal{P}\}.$$

It is easy to see that the value $\rho(\mathcal{P})$ is always finite and the value $c_\rho(\mathcal{P})$ can be finite or infinite. For the chromatic number $\chi$ the invariant $\psi(\mathcal{P}) = \chi(\mathcal{P}) - 1$ is known as *subchromatic number* or *index* of the property $\mathcal{P}$ (cf. [2]). The value $c_\omega(\mathcal{P}) - 1$ if often called *completeness* of a property.

A graph theoretical invariant $\rho$ is called $\preceq$*-monotone* whenever for any pair $G_1, G_2$ of graphs satisfying $G_1 \preceq G_2$ holds $\rho(G_1) \leq \rho(G_2)$. A graph invariant $\rho$ is called *additive* whenever for any two graphs $G$ and $H$ holds the following: $\rho(G \cup H) = \max\{\rho(G), \rho(H)\}$.

**Example 2.** The properties $\mathcal{I}_k$, $\mathcal{O}^k$, $\mathcal{D}_k$, $\mathcal{S}_k$ and $\mathcal{O}_k$ mentioned above can be uniquely determined by the graph theoretical invariants $\omega(G)$ - the clique number, $\chi(G)$ - the chromatic number, $\mathrm{col}(G)$ - the coloring number (see [5]), $\Delta(G)$ - maximum degree and $o(G)$ - the order of the largest component of $G$. It is known that for any graph $G$ the following inequalities hold:

$$\omega(G) \leq \chi(G) \leq \mathrm{col}(G) \leq \Delta(G) + 1 \leq o(G).$$

It implies that $\mathcal{O}_k \subset \mathcal{S}_k \subset \mathcal{D}_k \subset \mathcal{O}^{k+1} \subset \mathcal{I}_k$. Moreover, it is easy to see that some other well-known invariants like choice number can be included into similar chains.

The following assertions follows almost immediately from the definitions.

Let $\preceq$ be a partial order on $\mathcal{I}$. Let $\rho$ be $\preceq$-monotone invariant and $\mathcal{P}$ be a $\preceq$-hereditary property. If $\mathcal{G}$ is a $\prec$-generating set of $\mathcal{P}$ then

$$c_\rho(\mathcal{P}) = \sup\{\rho(G) : G \in \mathcal{G}\}.$$

**Proposition 1.** *Let $\rho$ be an additive $\preceq$-monotone invariant and $\mathcal{P}, \mathcal{P}^*$ be $\preceq$-hereditary properties. Then for the meet and the join of the properties $\mathcal{P}$ and $\mathcal{P}^*$ (in the lattice $\mathbb{L}^a_\preceq$ of additive $\preceq$-hereditary properties) holds the following:*

(i) $\rho(\mathcal{P} \wedge \mathcal{P}^*) = \min\{\rho(\mathcal{P}), \rho(\mathcal{P}^*)\}$;

(ii) $\rho(\mathcal{P} \vee \mathcal{P}^*) \geq \max\{\rho(\mathcal{P}), \rho(\mathcal{P}^*)\}$;

(iii) $c_\rho(\mathcal{P} \vee \mathcal{P}^*) = \sup\{c_\rho(\mathcal{P}), c_\rho(\mathcal{P}^*)\}$;

(iv) $c_\rho(\mathcal{P} \wedge \mathcal{P}^*) \leq \inf\{c_\rho(\mathcal{P}), c_\rho(\mathcal{P}^*)\}$;

From the previous result it follows that, if two properties have finite values of the invariant $c_\rho$ then their intersection has finite value of $c_\rho$, too. Moreover, it is not difficult to see that the properties with finite value of $c_\rho$ forms an ideal of the lattice of additive $\preceq$-hereditary properties. In the case of the intersection of two properties with infinite characteristic $c_\rho$ the situation is much more complicated.

Consider a $\preceq$-monotone graph theoretical invariant $\rho$. It is not difficult to verify that for any non-negative integer $k$ the property $\mathcal{P}_{(\rho,k)} = \{G \in \mathcal{I} : \rho(G) \leq k\}$ is $\preceq$-hereditary. Moreover, if $\rho$ is also additive then the property $\mathcal{P}_{(\rho,k)}$ is additive too.

On the other hand, if $\mathcal{P}_0 \subseteq \mathcal{P}_1 \subseteq \mathcal{P}_2 \subseteq \cdots$ is a chain of $\preceq$-properties and $s_0$ is a non-negative integer, then we can define a new graph theoretical invariant $\rho$ associated to this chain in the following way: for any graph $G \in \mathcal{P}_0$ let us put $\rho(G) = s_0$ and for an arbitrary positive integer $s$ let $\rho(G) = s_0 + s$ if and only if $G$ belongs to $\mathcal{P}_s \setminus \mathcal{P}_{s-1}$. In addition, if the properties $\mathcal{P}_0, \mathcal{P}_1, \mathcal{P}_2, \ldots$ are additive then the defined invariant $\rho$ is additive too. In many cases it is natural and useful to put the initial value $s_0$ equal to zero. One can easily observe that if for some non-negative integer $k$ we have the equality $\mathcal{P}_k = \mathcal{P}_{k+1}$ then the defined invariant does not attain the value $k+1$ for any graph $G \in \mathcal{I}$. Moreover, if there is an index $k_0$ such that for any $k > k_0$ the equality $\mathcal{P}_{k_0} = \mathcal{P}_k$ holds then

| $\mathcal{P}$ | $\mathcal{O}$ | $\mathcal{O}_k$ | $\mathcal{S}_k$ | $\mathcal{W}_k$ | $\mathcal{D}_k$ | $\mathcal{O}^{k+1}$ | $\mathcal{I}_k$ |
|---|---|---|---|---|---|---|---|
| $\omega$ | 2 | 2 | 2 | 2 | 2 | 2 | k+2 |
| $c_\omega$ | 1 | k+1 | k+1 | k+1 | k+1 | k+1 | k+1 |
| $\chi$ | 2 | 2 | 2 | 2 | 2 | k+2 | k+2 |
| $c_\chi$ | 1 | k+1 | k+1 | k+1 | k+1 | k+1 | $\infty$ |
| col | 2 | 2 | 2 | 2 | k+2 | k+2 | k+2 |
| $c$ col | 1 | k+1 | k+1 | k+1 | k+1 | $\infty$ | $\infty$ |
| $\Delta+1$ | 2 | 3 | k+2 | 3 | k+2 | k+2 | k+2 |
| $c_{\Delta+1}$ | 1 | k+1 | k+1 | $\infty$ | $\infty$ | $\infty$ | $\infty$ |

Table 1: Values of $c_\rho$ and $\rho$ for some examples of hereditary properties

the invariant associated to this chain gets value at most $k_0$ for an arbitrary graph $G \in \mathcal{I}$.

**Proposition 2.** *Let $\mathcal{P}$ be a $\preceq$-hereditary property and $\rho$ be a graph theoretical invariant. Then $c_\rho(\mathcal{P})$ is finite if and only if there exists a positive integer $k$ such that $\mathcal{P} \subseteq \mathcal{P}_{(\rho,k)}$.*

**Example 3.** Let $\mathcal{B}_i$, $i = 0, 1, 2, \ldots$ be hereditary properties defined in terms of their set of forbidden subgraphs in the following way: $\mathrm{F}(\mathcal{B}_i) = \{K_{m,n} : m+n = i+2\}$. Then is clear that they form a chain of the lattice $\mathbb{L}^a$ and for any graph $G \in \mathcal{I}$ the invariant associated to this chain determines the order of the largest complete bipartite graph contained in $G$.

Some more details and many other examples on the relationship between invariants will be presented. The talk is based on an unpublished paper by M. Jacobson, P. Mihók and G. Semanišin.

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## DOMINATION AND INDEPENDENCE IN GRAPH PRODUCTS

Douglas F. Rall
*Furman University, Greenville, USA*

The study of how graphical parameters act on graph products has provided some of the most difficult open problems in graph theory. Classical examples from coloring, independence and domination are Hedetniemi's conjecture, Shannon capacity and Vizing's conjecture.

A graphical parameter $\sigma$ is said to be *multiplicative* with respect to the graph product $\otimes$ if one of the following holds for all graphs $G$ and $H$

- $\sigma(G \otimes H) \geq \sigma(G)\sigma(H)$,
- $\sigma(G \otimes H) \leq \sigma(G)\sigma(H)$.

In this talk we consider whether some of the domination and independence parameters are multiplicative with respect to several of the common graph products (e.g., Cartesian, categorical). In addition, we report on recent work related to finding bounds on $\sigma(G \otimes H)$ in terms of $\sigma(G)\sigma(H)$ in these same contexts.

**Keywords:** graph products, domination, independence, multiplicative.
**AMS Subject Classification:** 05C69, 05C70.

## RAMSEY AND RAINBOW COLOURINGS

Ingo Schiermeyer

*Freiberg University of Mining and Technology, Freiberg, Germany*

In this talk we consider edge colourings of graphs. For given graphs $F_1, F_2, \ldots, F_k, k \geq 2$, the *Ramsey number* $r(F_1, \ldots, F_k)$ is the smallest integer $n$ such that if we arbitrarily colour the edges of the complete graph of order $n$ with $k$ colours, then there is always a monochromatic copy of some $F_i$ for $1 \leq i \leq k$. We will list the Ramsey numbers if the graphs $F_i$ are complete or cycles and report about recent progress on some conjectures of Erdős ([2], [3]).

For given graphs $G, H$ the *rainbow number* $rb(G, H)$ is the smallest number $m$ of colours such that if we colour the edges of $G$ with at least $m$ different colours, then there is always a totally multicoloured or rainbow copy of $H$. For various graph classes of $H$ we will list the known rainbow numbers if $G$ is the complete graph [1] and report about recent progress on the conjecture of Erdős, Simonovits and Sós on the rainbow numbers $rb(K_n, C_k)$ for cycles. Finally, new results on the rainbow numbers $rb(Q_n, Q_2)$ for the hypercube $Q_n$ will be presented.

**Keywords:** edge colouring, Ramsey, rainbow, extremal graphs.
**AMS Subject Classification:** 05C15, 05C35.

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## ON GENERALIZED $k$-DEGENERATE GRAPHS

Gabriel Semanišin
*P.J. Šafárik University, Košice, Slovak Republic*

Let $k$ be a non-negative integer. A graph is called *$k$-degenerate* if the minimum degree of each its subgraph is at most $k$. The basic characterization of $k$-degenerate graphs can be found in [2]. The degree sequences of $k$-degenerate graphs were characterized in [1].

The property "to be a $k$-degenerate graph" is additive and hereditary and plays an important role in the lattice of additive induced-hereditary properties of graphs. Its position in the lattice was described in [3] and [4].

Let $\mathcal{P}$ be a hereditary property of graphs, let $G$ be a graph and let $v$ be a vertex of $G$. We introduce a $\mathcal{P}$-degree of $v$ in the following way:

$$\begin{aligned}\deg_G^{\mathcal{P}}(v) \quad = \quad & \max\{k : \text{there exist sets } V_1 \cup V_2 \cup \cdots \cup V_k \subseteq V(G) \text{ s.t. for each } i \\ & G[V_i] \cong F \text{ for some } F \in \mathbf{F}(\mathcal{P}) \text{ and } V_i \cap V_j = \{v\} \text{ for each } i \neq j\},\end{aligned}$$

where $\mathbf{F}(\mathcal{P})$ is the set of minimal forbidden subgraphs (in the case of the ordinary vertex degree we have only one forbidden graph, namely $K_2$).

Using this invariant we can define the property "to be $k$-degenerate with respect to $\mathcal{P}$" and discuss its position in the lattice of additive induced-hereditary properties of graphs.

**Keywords:** $k$-degenerate graph, generalized colouring, hereditary property, minimal reducible bound.
**AMS Subject Classification:** 05C15.

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## TREES WITH NUMEROUS EXTREMAL SUBFORESTS

Zdzisław Skupień
*AGH University of Science and Technology, Kraków, Poland*

A review of results, old and new, on the structure of $n$-vertex trees with maximal numbers of some selected extremal subforests will be presented.

These subforests are some factors (e.g., maximum linear forests), maximal matchings, maximal independent sets, or kernels with various distance bounds.

**Keywords:** tree, independent set, kernel, maximizing cardinality, structure.
**AMS Subject Classification:** 05C05, 05C69, 05C35, 05C75, 05A16, 39A10.

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## ON SELF-COMPLEMENTARY SUPERGRAPHS OF $(N, N)$-GRAPHS

A. Paweł Wojda, Mariusz Woźniak and Irmina A. Zioło
*AGH University of Science and Technology, Kraków, Poland*

We consider simple graphs without loops and multiple edges.

An *embedding* of a graph $G$ is a permutation $\sigma$ on $V(G)$ such that if an edge $xy$ belongs to $E(G)$ then $\sigma(x)\sigma(y)$ does not belong to $E(G)$. If there exists an embedding of $G$ we say that $G$ is *embeddable*.

A graph $G$ of order $n \equiv 0, 1 \pmod 4$ is *self-complementary* if it is isomorphic to its complement. A graph $G$ of order $n \equiv 2, 3 \pmod 4$ is *almost self-complementary* if $G$ is of size $\frac{1}{2}(\binom{n}{2}-1)$ and $G$ is a subgraph of its complement.

It is evident that subgraphs of self-complementary graphs are embeddable. In general the converse is not true. It is proved that every embeddable graph of order $n$ and size at most $n-1$ is a subgraph of self-complementary or almost self-complementary, respectively, graph of order $n$ ([1] cases $n \equiv 0, 1 \pmod 4$, [2] cases $n \equiv 2, 3 \pmod 4$). We prove that, with one exception, for each embeddable graph of order $n$ and size $n$ there exists a self-complementary or an almost self-complementary, respectively, supergraph of order $n$.

**Keywords:** packing of graphs, self-complementary graph.
**AMS Subject Classification:** 05C70.

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## ON THE SPLIT DOMINATION NUMBER OF THE CARTESIAN PRODUCT OF PATHS

Maciej Zwierzchowski

*Technical University of Szczecin, Poland*

Let $D$ be a dominating set of $G$. If the subgraph induced by the subset $V(G) - D$ is disconnected, then $D$ is called a *split dominating set* of $G$. By $\gamma_s(G)$ we mean the cardinality of the smallest split dominating set of $G$ and we call it the *split domination number* of $G$. The concept of split domination comes from [2].

In this paper we discuss the split domination number with respect to the Cartesian product of paths. Motivation of this problem comes from [1], where was study the domination number of $P_m \times P_n$. We calculate the $\gamma_s(P_2 \times P_n)$ and estimate the $\gamma_s(P_m \times P_n)$ using the domination number of $P_m \times P_n$. Further, we discuss a $\gamma_s(P_m \times P_n)$ with respect to large integer $m$ and $n$.

**Keywords:** domination number, split domination number, Cartesian product of graphs.

**AMS Subject Classification:** 05C69.

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# LIST OF PARTICIPANTS


| | |
|---|---|
| Simson Agu | gomes_fundation@yahoo.com |
| Krystyna T. Balińska | balinska@man.poznan.pl |
| Małgorzata Bednarska | mbed@amu.edu.pl |
| Halina Bielak | hbiel@golem.umcs.lublin.pl |
| Jacek Bojarski | j.bojarski@im.uz.zgora.pl |
| Mieczysław Borowiecki | m.borowiecki@im.uz.zgora.pl |
| Piotr Borowiecki | p.borowiecki@im.uz.zgora.pl |
| Boštjan Brešar | bostjan.bresar@uni-mb.si |
| Izak Broere | ib@rau.ac.za |
| Krzysztof Bryś | brys@alpha.mini.pw.edu.pl |
| Stanisław Bylka | bylka@ipipan.waw.pl |
| Ewa Drgas-Burchardt | e.drgas-burchardt@im.uz.zgora.pl |
| Tomasz Dzido | tdz@math.univ.gda.pl |
| Zyta Dziechcińska-Halamoda | halamoda@math.uni.opole.pl |
| Anna Fiedorowicz | a.fiedorowicz@im.uz.zgora.pl |
| Hanna Furmańczyk | hanna@math.univ.gda.pl |
| Izolda Gorgol | gorgol@antenor.pol.lublin.pl |
| Małgorzata Grajdek | kosma@amu.edu.pl |
| Harald Gropp | d12@ix.urz.uni.heidelberg.de |
| Jarosław Grytczuk | j.grytczuk@im.uz.zgora.pl |
| Mariusz Hałuszczak | m.haluszczak@im.uz.zgora.pl |
| Jochen Harant | harant@mathematik.tu-ilmenau.de |
| Michael A. Henning | henningm@mail.etsu.edu |
| Mirko Horňák | hornak@duro.science.upjs.sk |
| Adam Idzik | adidzik@ipipan.waw.pl |
| Stanislav Jendrol' | jendrol@kosice.upjs.sk |
| Konstanty Junosza-Szaniawski | junoszak@prioris.mini.pw.edu.pl |
| Mihyun Kang | kang@informatik.hu-berlin.de |
| Hal Kierstead | kierstead@asu.edu |
| Sandi Klavžar | sandi.klavzar@uni-mb.si |
| Marián Klešč | marian.klesc@tuke.sk |
| Magdalena Kucharska | magdakucharska@wp.pl |
| Maria Kwaśnik | kwasnik@arcadia.tuniv.szczecin.pl |
| Anna B. Kwiatkowska | aba@mat.uni.torun.pl |

| | |
|---|---|
| Magdalena Lemańska | magda@mifgate.mif.pg.gda.pl |
| Tomasz Lubiński | tomasz_lubinski@o2.pl |
| Ewa Łazuka | elazuka@antenor.pol.lublin.pl |
| Daniel Łocman | locman@post.pl |
| Zofia Majcher | majcher@math.uni.opole.pl |
| Jerzy Michael | michael@uni.opole.pl |
| Danuta Michalak | d.michalak@im.uz.zgora.pl |
| Peter Mihók | peter.mihok@tuke.sk |
| Monika Perl | mperl@arcadia.tuniv.szczecin.pl |
| Monika Pszczoła | monikafr@mini.pw.edu.pl |
| Douglas F. Rall | drall@herky.furman.edu |
| Ingo Schiermeyer | schierme@math.tu-freiberg.de |
| Gabriel Semanišin | semanisin@science.upjs.sk |
| Elżbieta Sidorowicz | e.sidorowicz@im.uz.zgora.pl |
| Zdzisław Skupień | skupien@uci.agh.edu.pl |
| Maciej M. Sysło | syslo@ii.uni.wroc.pl |
| Alina Szelecka | a.szelecka@im.uz.zgora.pl |
| Jerzy Topp | topp@mif.pg.gda.pl |
| Zsolt Tuza | tuza@sztaki.hu |
| Bartosz Wiszniewski | b.wiszniewski@im.uz.zgora.pl |
| A. Paweł Wojda | wojda@uci.agh.edu.pl |
| Adam Wysoczański | a.wysoczanski@im.uz.zgora.pl |
| Irmina A. Zioło | ziolo@uci.agh.edu.pl |
| Maciej Zwierzchowski | mzwierz@ps.pl |

## A COMPLETE PROOF OF A HOLYER PROBLEM

Krzysztof Bryś and Zbigniew Lonc
*Warsaw University of Technology, Poland*

In this paper we deal with so-called edge decompositions of graphs. A set of graphs $\{G_1, ..., G_s\}$ is called a *decomposition* of a graph $G$ if $E(G_1) \cup ... \cup E(G_s) = E(G)$ and $E(G_i) \cap E(G_j) = \emptyset$, for $i \neq j$. Let $H$ be a graph. An $H$-*decomposition* is a decomposition $\{G_1, G_2, ..., G_s\}$ of $G$ such that each $G_i$ is isomorphic to $H$. For vertex disjoint graphs $G$ and $H$ let $G \cup H$ be their union and let $pH$ be the disjoint union of $p$ copies of $H$. Denote by $P_k$ the $k$-vertex path.

Holyer [3] raised the problem of establishing the complexity status of the problem $\mathcal{P}_H$ of existence of an $H$-decomposition of a graph $G$, where $H$ is not a part of the instance. He proved the NP-completeness of the problem $\mathcal{P}_H$ for complete graphs of order at least 3, for paths $P_k$, $k \geq 4$ and for cycles.

Alon [1] proved polynomiality of the problem $\mathcal{P}_H$ when $H = sP_2$, for any fixed positive integer $s$. Priesler and Tarsi [5] showed that $\mathcal{P}_H$ is polynomial when $H = P_3 \cup tP_2$ and $t$ is any fixed positive integer.

Dor and Tarsi [2] proved a strong result that the problem $\mathcal{P}_H$ is NP-complete whenever $H$ contains a connected component with at least 3 edges. They conjectured that $\mathcal{P}_H$ is polynomial in the remaining cases, i.e. when $H = sP_3 \cup tP_2$. Lonc [4] proved polynomiality of $\mathcal{P}_H$ for $H = sP_3$.

In this contribution we solve completely the problem of classifying the problems $\mathcal{P}_H$ according to their computational complexity by proving the following theorem.

**Theorem 1.** *The problem $\mathcal{P}_H$, where $H = sP_3 \cup tP_2$, is polynomial.*

We have found a polynomial algorithm checking if $G$ admits an $sP_3 \cup tP_2$-decomposition.

### References

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## SOME FAMILIES OF POSETS OF PAGE NUMBER 2

Anna B. Kwiatkowska
*Mikołaj Kopernik University, Toruń, Poland*

and
Maciej M. Sysło
*University of Wrocław, Poland*

The *embedding of a graph G in a book* consists of placing the vertices of $G$ on the spine in some order and assigning each edge of $G$ to one of the pages in such a way, that the edges assigned to one page do not cross. The *page number* of $G$ is the smallest number $k$ such that $G$ has a book embedding on k pages.

In some applications, orderings of vertices on the spine are restricted to linear extensions of a partially ordered set $P$. In such case one may define the page number of $P$.

Several lower and upper bounds to the page number of a poset $P$ have been established, e.g. in the terms of the clique number and vertex covers of the digram of $P$, and also in the terms the jump number of $P$. It follows from the latter bound that the page number of $P$ is 1 iff the diagram of P has no cycle. We shall use the same bound to provide families of posets with page number 2. The problem of complete characterization of such posets seems to be very difficult. Its complexity status is still open and it is known that the recognition problem of graphs with page number 2 is NP-complete.